MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SECOND SEMESTER EXAMINATIONS FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE (CIVIL ENGINEERING)

ECU 202: ENGINEERING MATHEMATICS VII

DATE: 19/4/2023 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS TO CANDIDATES.

Answer question one (**compulsory**) and any other **two** questions

QUESTION ONE (30 MARKS)

a) The resonance frequency f_r in a series electrical circuit is given by, $f_r = \frac{1}{2\pi\sqrt{LC}}$, evaluate $\frac{\partial f_r}{\partial L}$ (1 marks)

b) Solve the differential equation $\frac{dy}{dx} = 2e^x + \sin x$ (3 marks)

c) Show that $y = Ae^x + Be^{-x} + Ce^{2x}$ is the solution of the differential equation (4 marks)

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0$$

d) Sketch the family of curves represented by the differential equation $\frac{dy}{dx} = 2x + 3$ and determine the curve passing through $(1, 3)$ (5 marks)

e) The current i in an electrical circuit is given by the differential equation $Ri + L\frac{di}{dt} = 0$ where R is the electrical resistance and L inductance. If at $i = I$ at $t = 0$ show that the current at any given time is given by $i = Ie^{\frac{-Rt}{L}}$ (4 marks)

f) Show that the differential equation $(x^2 - 3y^2)dx + 2xydy = 0$ is homogeneous and hence find its solution. (4 marks)

g) Find the orthogonal trajectory of the family of curves defined by $x^2 + 2y^2 = C$.(3 marks)

h) Solve the following initial value problems.

(i) $6\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 6y = 0$, $y(0) = 5$ and $y'(0) = 1$ (3 marks)

(ii) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 13y = 0$, $y(0) = 4$ and $y'(0) = 0$ (3 marks)

QUESTION TWO (20 MARKS)

a) Solve the differential equation $\frac{dy}{dx} = \frac{2x+3y-8}{5x+y-7}$ (5 marks)

b) Show that the differential equation $(x - xy^2)dx + (8y - x^2y)dy = 0$ is exact and hence solve it. (4 marks)

c) Show that the solution of the differential equation $\frac{dy}{dx} - 2(x+1)^3 = \frac{4}{x+1}y$ is

$y = (x+1)^4 \ln(x+1)^2$, given that at $y(0) = 0$ (5 marks)

d) The charge q of a capacitor in a certain electrical circuit satisfies the differential equation $\frac{d^2q}{dt^2} + 4\frac{dq}{dt} + 5q = 0, q(0) = Q$ and $q'(0) = 0$, show that the charge in the circuit can be expressed as $q = \sqrt{5}Qe^{-2t}\sin(t + 0.464)$. (6 marks)

QUESTION THREE (20 MARKS)

a) Consider the differential equation $2x\ln x\frac{dy}{dx} + y = 0$

(i) Show that the differential equation is not exact. (3 marks)

(ii) Show that $\frac{y}{x}$ is an integrating factor of the differential equation. (4 marks)

(iii)Using the integrating factor, solve the differential equation. (5 marks)

b) A string of length 50 cm is set to oscillate by displacing its mid-point a distance of 2 cm from its rest position and released with zero velocity. Solve the wave equation $\frac{\partial^2u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2u}{\partial t^2}$ at $c^2 = 1$ and hence determine the resulting motion $u(x,t)$. (8 marks)

QUESTION FOUR (20 MARKS)

a) The potential difference V between the plates of a capacitor of capacitance C charged by a steady voltage E through a resistor is given by $CR\frac{dV}{dt} + V = E$

(i) Solve the equation for V given that at $t = 0. V = 0$ (7 marks)

(ii) Calculate V correct to 3 significant figures when $E = 25V, C = 20 \times 10^{-6}F, R = 200 \times 10^{3}\Omega$ and $t = 3s.$ (3 marks)

b) Solve the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = e^{2x} + 3x^2 + 4x - 5 + 5\cos 2x.$

(10 marks)

QUESTION FIVE (20 MARKS)

a) Solve the differential equations

(i) $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 10y = e^{2x} + 20, y(0) = 0, y'(0) = \frac{-1}{3}$ (5 marks)

(ii) $\frac{d^2s}{dt^2} - 6\frac{ds}{dt} + 9s = 0\ , s(0) = 2, s'(0) = 0$ (5 marks)

b) Solve the system (10 marks)

$$(D + 2)x + 3y = 0$$

$$3x + (D + 2)y = 2e^{2t}$$