MACHAKOS UNIVERSITY

University Examinations for 2018/2019

SCHOOL OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

FIFTH YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)

EEE 512: DIGITAL SIGNAL PROCESSING

DATE: **TIME:**

INSTRUCTIONS

This paper consists of **FIVE** questions. Answer Question **ON**E and any other **TWO** questions

QUESTION ONE (COMPULSORY-30 marks)

a) Given two sequences

$$x_1[n] = 3\delta[n] + 2\delta[n-1]$$
$$x_2[n] = \delta[n-1] + 2\delta[n]$$

i. Find the Z-transform of their convolution $Z\{x_1[n] + x_2[n]\}$

ii. Determine the convolution using Z-transforms

(5 marks)

b) Find the inverse Z-transform of

$$H(z) = \frac{10z}{z^2 + z + 1}$$

(3 marks)

c) (c) A discrete system is described by the difference equation $y[n] = 3x[n] + 5$. Determine whether the system is

i. linear

ii. time-invariant

(6 marks)

d) (i)Distinguish between decimation in time and decimation if frequency FFT algorithms

(ii)Explain how pipelining improves the operation of a DSP processor.

(4 marks)

e) Given a sequence $x[n]$ for $0 \leq n \leq 3$ where $x(0) = 1, x(1) = 2, x(2) = 3, x(4) = 4$. Evaluate its DFT X(k).

(4 marks)

f) A discrete-time processing operation is defined by the recurrence equation

$$\mathrm{y[n] = x[n] + 2x[n-1] + 3x[n-3]}$$

Determine the unit sample response $h[n]$ of the processor

(3 marks)

g) Given a second order transfer function

$$H(z) = \frac{0.5(1 - z^{-2})}{1 + 1.3z^{-1} + 0.36z^{-1}}$$

Perform the filter realization and write the difference equations using the following realizations:

i. Direct I form
ii. Direct II form

(5 marks)

QUESTION TWO (20 Marks)

a) A signal is to be digitally filtered using a digital filter based on an analogue second-order Butterworth filter with a normalized transfer function

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

The signal sampling frequency is 2Khz and the filter cut-off frequency is 600Hz.

i. Using bilinear transformation, obtain the transfer function of the digital filter
ii. Draw a realization of the filter structure using minimum number of delay elements

(10 marks)

b) The transfer function of an LTI system is

$$H(z) = \frac{z}{(z - 0.75)(z + 0.5)}$$

I. For each possible region of convergence, find the corresponding impulse response of the system
II. State the ROC where the system is causal and the ROC for which it is stable

(10 marks)

QUESTION THREE (20 MARKS)

a) Distinguish between decimation-in-time and decimation in frequency as used in FFT (2 marks)

b) (b)(i)Starting from the from the definition of the DFT of $X[k] = \sum_0^{N-1} W_N^{nk}$, and using a 4-point DFT, show that the transform can be expressed as $\overline{\boldsymbol{X}}[\boldsymbol{k}] = \boldsymbol{W}\boldsymbol{x}[\boldsymbol{n}]$, where $\overline{\boldsymbol{X}}[\boldsymbol{k}]$ and $\boldsymbol{x}[\boldsymbol{n}]$ are column matrixes and $\boldsymbol{W}$ a transformation matrix.

(ii)Using the transformation Matrix in (i), determine the DFT of the sequence $x(n) = \{\underset{\uparrow}{1}, 2,2,1\}$

(10 marks)

(b)By means DFT and IDFT, determine the response of the FIR filter with impulse response

$$h(n) = \{\underset{\uparrow}{1}, 2, 3\}$$

to the input sequence

$$x(n) = \{\underset{\uparrow}{1}, 2,2,1\}$$

(8 marks)

QUESTION THREE (20 MARKS)

(a) Given that $h(n) = \{1,0,1\}$ and $h(n) = \{1, 3, 2, -3, 0, 2, -1, 0, -2, 3, -2, 1, ..\}$, convolve the two sequences by using the following methods
   i. Overlap and add;
   ii. Overlap and save;

(10 marks)

(b)Given the filter

$$H(z) = \frac{1 - 0.9z^{-1} - 0.1z^{-2}}{1 + 0.3z^{-1} - 0.04z^{2}}$$

Realize $H(z)$ and develop the difference equation using the following form:
   i. Cascaded (series)form via first
   ii. Parallel form via first order-sections

(10 marks)

QUESTION FOUR (20 MARKS)

a) An analogue prototype transfer function $H(s) = \frac{s}{s-0.5}$ is used to find an equivalent low pass digital whose gain is -3dB at 400Hz and sampling frequency 4 KHz. Using bilinear transform
   i. Derive the transfer function of the digital filter
   ii. Find the difference equation relating its output to the input sequence

(12 marks)

b) A linear-phase filter has a double zero at z=-1 and a conjugate pair of zeros at $z = -1$ and a conjugate pair of zeros at $0.5(1 + j\sqrt{3})$. Derive a stable, causal real impulse response $h(n)$ for this filter. Assume that the d.c gain is 1.

(8 marks)

QUESTION FIVE (20 MARKS)

a) (i)Explain how circular convolution of two sequences can be obtained using FFTs and IFFs only

(ii)Using the method in (i) and a radix-2 decimation in time FFT algorithm find the circular convolution of the sequences.

$$X_1[n] = [3,1,2,5]$$
$$X_2[n] = [1,2,0,2]$$

(10 marks)

b) Convert the analog filter $H(s) = \frac{1}{(s+1)(s+2)}$ into digital filter using the impulse invariant technique with a sampling interval of 0.02s.

(10 marks)