MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

FOURTH SUPPLEMENTARY EXAMINATION FOR BACHELOR OF EDUCATION (SCIENCE)

SPH 400: CLASSICAL MECHANICS

DATE: 24/7/2019 **TIME: 11:00 – 1:00 PM**

INSTRUCTIONS:

- The paper consists of **two** sections.
- Section **A** is **compulsory** (30 marks).
- Answer any **two** questions from section **B** (each 20 marks).

Required data and constants:

Earth's gravitational pull = 9.8 N/Kg

The Universal Gravitational Constant $G = 6.67 \times 10^{-11} N.m^2/Kg^2$

SECTION A (COMPULSORY)

QUESTION ONE

a) State Hamilton's Principle (2 marks)

b) (i) Discuss why is not necessary for generalized coordinates to have dimensions of length. (2 marks)

(ii) State the mathematical conditions that must be satisfied by Generalized coordinates (2 marks)

c) Shown that if there is no external force acting on two bodies colliding elastically then the total momentum of the system is converted. (5 marks)

d) State three application of Lagrange's equation for motion of various mechanical system (3 marks)

e) Distinguish the following terms:

(i) Discrete and continuous systems (2marks)

(ii) Constraints and Degrees of freedom (2marks)

(iii) Rigid and elastic bodies (2marks)

f) As an example of recoil, let us consider recoil in radioactive decay. An alpha particle, which is the nucleus of the He-atom, is emitted from a Uranium-238 nucleus, originally at rest, with a speed of 1.4 x $10^7 ms^{-1}$ and a kinetic energy of 4.1MeV. Find the recoil speed of the residual nucleus which is Thorium-234. (5marks)

g) (i) With examples, state the constraints involved in solving Newtonian mechanics problems (3 marks)

(ii) How are those constraints eliminated (2 marks)

QUESTION TWO

a) Two skaters collide and embrace each other. One has mass m_1 = 70kg and is initially moving east at a speed u_1 = 6km/h while the other has mass m_2 = 50kg and is initially moving north at a speed u_2 = 8km/h.

i) What is the final velocity of the couple? (5 marks)

ii) What fraction of the initial kinetic energy is lost because of the collision (5 marks)

b) From linear momentum of system of particles show that $\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + ... + \vec{p}_n =$ constant. Where the symbols carry their usual meaning (10 marks)

QUESTION THREE

a) Use Hamilton's Principle to find the equation of motion of a one-dimensional harmonic oscillator. (10 marks)

b) Find the velocity of a rocket moving vertically upward at any time t in a constant gravitational field. Assume that the gas is ejected at a constant rate and at a constant velocity with respect to the rocket. (10 marks)

QUESTION FOUR

a) Using Lagrange's equations, for a system of single particles, with coordinate $\boldsymbol{X}_i$ (where i= 1, 2, 3). Show the eguivalence of Lagranges's and Newton' equations (6 marks)

b) A rocket is moving away from the solar system at a speed of 6.0 x 10^3ms^{-1}. It fires its engine which ejects exhaust with a relative velocity of 3.0 x 10^3ms^{-1}. The mass of the rocket at this time is 4.0 x 10^4kg and it experiences an acceleration of 2.0ms^{-2}. Find

i) the velocity of the exhaust relative to the solar system (7 marks)

ii) the rate at which the exhaust was ejected during the firing. (7 marks)

QUESTION FIVE

a) Let a mass *m* be attached to a spring of force constant *k*. Obtain its equation of motion and frequency of oscillation using Lagrange equations. (8 marks)

b) Consider a circular disc of mass *M* and radius *R* rolling down an inclined plane without slipping. Find the speed of its center of mass when it reaches the bottom of the incline. (12 marks)