MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

SMA 361-THEORY OF ESTIMATION

Date: **Time:**

ANSWER QUESTION ONE AND OTHER TWO QUESTIOS

Question 1 (Compulsory)

(a) Explain the meaning of the following terms as applied in theory of estimation

(i) Population
(ii) An estimator (2marks)

(b) A random sample of five values is taken from a normal population with mean μ and variance δ^2 both unknown. The values are: 1.07, 1.02, 1.04, 1.06, and 1.05. Estimate the population mean and population variance. (3 marks)

(c) $Let\ x_1, x_2, .. x_n$ be identically independent random variable with a finite mean μ,show that the sample mean $\bar{x}$ is always unbiased estimator of μ. (4marks)

(d) Let $x_1, x_2, .. x_n$ denote a random sample from a pdf is $f(x, \theta) = \theta x^{\theta-1}$,0$< x < 1, \theta > 0$

Show that $t_1 = \prod_{i-1}^{n} x_i$ is sufficient for θ (4marks)

(e) $Let\ x_1, x_2, ..x_n$ be identically independent random variable with a finite variance δ^2 .Show that the sample variance $s^2 = \frac{1}{n-1}(\sum_1^n (x-\bar{x})^2 = \frac{1}{n-1}\{\sum_1^n x^2 - n\bar{x}^2\}$ (5marks)

(f).Let $x_1, x_2, ..x_n$ be a random sample of size n from a distribution with mean from a population with mean μ (unknown) and known variance δ^2.Determine the maximum likelihood estimator (MLE) of δ^2 . (6marks)

(g).Given a population with a distribution whose pdf is

$$f(x,\theta) = \frac{1}{\theta}, 0 \leq x \leq \theta, 0\ otherwise$$

Using the method of moments obtain the estimator for θ. (6marks)

Question Two

Three objects M, N and P with weights$t_{1,} t_2\ and, t_3$ respectively are weighed on a spring balance M and N together weigh 75g, M and P weigh 45g and N and P weigh 100g. Assuming the weights are independent with constant variance δ^2, determine the least square estimates of $t_1, t_2, and\ t_3$ (20 marks)

Question Three

(a) Let $x_1, x_2, ..x_n$ be a random sample from $x \sim N(\mu_0, \delta^2)$ where μ, δ^2 are both unknown. Determine a joint sufficient statistic of $\mu, and\ \delta^2$ (8 marks)
(b) A random sample of size n is drawn from a normal population with $(0,\delta^2)$. Determine the minimum variance unbiased estimator (MVUE) of δ^2 and its variance. (12 marks)

Question Four

(a) A population has a known mean μ and a standard deviation δ of 2.45. If a sample of 900 has a mean of 3.15cm and the population is normal obtain a 95% confidence intervals for the true mean. (10 marks)

(b) Let X be random variable that is binomially distributed with a pdf defined as

$f(x;\theta) = \binom{n}{x}\theta^x(1-\theta)^{n-x}$ $for\ x = 0,1 \ldots n, \theta \in (0,1)$.Determine the Crammer-Rao lower bound (10marks)

Question Five

(a) Let $x_1, x_2, .. x_n$ be a random variable of size n from a population which is normally distributed with mean μ and δ^2 both unknown. Determine the MLE of μ and δ^2 (8marks)

(b) A random sample $(X_1, X_2, X_3, X_4, X_5)$ of size 5 is drawn from a normal population with unknown mean μ. Consider the following estimators to estimate μ

$$t_1 = \frac{X_1 + X_2 + X_3 + X_4 + X_5}{5}$$

$$t_2 = \frac{X_1 + X_2}{2} + X_3$$

$$t_3 = \frac{2X_1 + X_2 + \lambda X_3}{3}$$

Where λ is such that t_3 is an unbiased estimator of μ

(i) Determine λ (4marks)

(ii) Show that $t_1 and\ t_2$ are unbiased (4marks)

(iii) State giving reasons the estimators which is best among $t_1, t_2, and\ t_3$ (4marks)