# MACHAKOS UNIVERSITY

**University Examinations for 2021/2022 Academic Year**

**SCHOOL OF PURE AND APPLIED SCIENCES**

**DEPARTMENT OF MATHEMATICS AND STATISTICS**

**SECOND YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR**

**BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONIC ENGINEERING)**

**BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)**

**BACHELOR OF SCIENCE (CIVIL ENGINEERING)**

**ECU 202: ENGINEERING MATHEMATICS VII**

**DATE: 26/8/2022** **TIME: 11.00-1.00 PM**

**INSTRUCTIONS**

Answer **question One (Compulsory)** and any other **Two questions**

## QUESTION ONE (30 MARKS)

a) Show that $y = x^2 + 4$ is a solution of the equation $3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 10x + 6$ (2 marks)

b) Obtain the differential equations associated with the following primitives

i. $y = Ae^{2x} + Be^{x} + C$ (3 marks)

ii. $y = Ae^{2x} + B$ (2 marks)

c) Solve the following equations

i. $\frac{dy}{dx} = \frac{xy^2+x}{yx^2+y}$ (5 marks)

ii. $(3x + 2y - 5)dx + (2x + 3y - 5)dy = 0$ (10 marks)

d) Using the substitution $y = vx$ transform the equation $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$ into an equation containing $v$ and $x$ only. Hence solve the resulting equation using method of separation of variables (8 marks)

## QUESTION TWO (20 MARKS)

a) Show that the equation $3x(xy - 2)dx + (x^3 + 2y)dy = 0$ is exact. Hence solve the equation (6 marks)

b) Solve $\frac{dy}{dx} + \frac{2x}{1+x^2} = \frac{1}{(1+x^2)^2}$ (4 marks)

c) Find solutions to the following equations

i. $\frac{dy}{dx} + y = xy^3$ (6 marks)

ii. $\frac{d^5y}{dx^5} - 2\frac{d^4y}{dx^4} + \frac{d^3y}{dx^3} = 0$ (4 marks)

**QUESTION THREE (20 MARKS)**

a) Solve $y'' \sin y' = \sin x$ which satisfies the condition $y(1) = 2$ and $y'(1) = 1$ (6 marks)

b) Solve the equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$ (4 marks)

c) The rate at which radioactive nuclei decay is proportional to the number of nuclei that are present in a given sample. Half of the original number of radioactive nuclei have undergone disintegration in a period of fifty years.

   i. Calculate the percentage of the original radioactive nuclei which will remain after one hundred and fifty years (6 marks)

   ii. In how many years will seventy five percent of the original number remain? (4 marks)

**QUESTION FOUR (20 MARKS)**

a) The population $x$ of a certain city satisfies the law $\frac{dx}{dt} = \frac{x}{100}$ where $t$ is measured in years. Given that the population of this city is 200000 in 1980;

   i. Find the equation connecting the population $x$ and time $t$. $(> 1980)$ (5 marks)

   ii. Determine in what year does the population double (5 marks)

b) A circuit has in series a constant electromotive force $40$ Volt, a resistor of $10\ \Omega$ and an inductance 0.2 Henry, the initial current is zero

   i. Sketch the circuit (2 marks)

   ii. State the basic equation in electric circuit (1 mark)

   iii. By solving the basic equation find the current, $i$ on the circuit at time $t > 0$ (5 marks)

   iv. What is the steady current after a long time? (2 marks)

**QUESTION FIVE (20 MARKS)**

a) Consider the differential equation $x^2y\frac{d^2y}{dx^2} + \left(x\frac{dy}{dx} - y\right)^2 - 3y^2 = 0$

   i. Show that the above equation is nonlinear (3 marks)

   ii. Solve the above nonlinear equation (12 marks)

b) Find a first integral of the following nonlinear differential equations

   i. $\left(y^2 + 2x^2\frac{dy}{dx}\right)\frac{d^2y}{dx^2} + 2(y + x)\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} + y = 0$ (3 marks)

   ii. $\frac{dy}{dx}\frac{d^2y}{dx^2} - x^2y\frac{dy}{dx} = xy^2$ (2 marks)