MACHAKOS UNIVERSITY

University Examinations for 2024/2025 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES
Department of Mathematics, Statistics and Actuarial Sciences

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR DEGREE IN BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE BACHELOR OF SCIENCE IN PROGRAMMING AND STATISTICS BACHELOR OF SCIENCE IN MATHEMATICS BACHELOR OF EDUCATION (SCIENCE) BACHELOR OF EDUCATION (ARTS)

SMA 432: PARTIAL DIFFERENTIAL EQUATIONS I

DATE: 16TH JANUARY, 2025 **TIME: 8.30 AM – 10.30 AM**

INSTRUCTIONS:
*Answer question **ONE (Compulsory)** and **any other TWO** questions*

QUESTION ONE *(COMPULSORY)* (30 MARKS)

a) Under what conditions can the Lagrange equation $Pp+Qq=R$ be classified as: Linear, Semi-Linear and Quasi-Linear. (4 marks)

b) Sketch a diagram to illustrate the notion of orthogonal trajectories on some surface $f(x,y,z)=0$ (2 marks)

c) Find a partial differential equation arising from $z=ax^2+by^2+ab$ (4 marks)

d) Find the complete solution to $(y-z)\frac{\partial z}{\partial x}+(x-y)\frac{\partial z}{\partial y}=z-x$ (5 marks)

e) Find the integral curves of the equations:

i) $\frac{dx}{x+y}=\frac{dy}{x+y}=\frac{dz}{-(x+y+2z)}$ (5 marks)

ii) $y^2zp-x^2zq=x^2y$ (5 marks)

iii) $\frac{dx}{xz-y}=\frac{dy}{yz-x}=\frac{dz}{1-z^2}$ (5 marks)

QUESTION TWO (20 MARKS)

a) Show Pfaffian differential equation given by $z\left(x+y^2\right)dx+z\left(z+x^2\right)-xy\left(x+y\right)dz=0$ is integrable. (7 marks)

b) Find the solution of the equation $\left(x^2-y^2-z^2\right)p+2xyq=2xz$ (9 marks)

c) Eliminate the arbitrary constant from $z=ax^2-by^3+4ab$ (4 marks)

QUESTION THREE (20 MARKS)

a) Given that $u \equiv u(x,y,z) = c_1$, $v \equiv v(x,y,z) = c_2$ are solutions of $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$. Show that $F(u,v) = 0$ is a general solution of the Lagrange's equation. (10 marks)

b) Find the integral curves of the equation $\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}$ (10 marks)

QUESTION FOUR (20 MARKS)

a) Consider a surface $F(x,y,z) = 0$ and let this surface intersect with the family of surfaces $G(x,y,z) = k$. Using Cramer's rule, derive the equations:
$P' = RF_y - QF_z, Q' = PF_z - RF_x, R' = QF_x - PF_y$ (10 marks)

b) Find the surface which intersects the surfaces of the system $z(x+y) = c(3z+1)$ and which passes through the circle $x^2 + y^2 = 1$, z = 1 (10 marks)

QUESTION FIVE (20 MARKS)

a) Determine the general solution to the P.D.E.
$xp + yq = 3z$ (6 marks)

b) Eliminate the arbitrary function from $\phi\left(x+y+z, x^2+y^2+z^2\right) = 0$ (4 marks)

c) Find the integral surface of the equation $x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$ which contains the straight line $x + y = 0$, $z = 1$ (10 marks)