MACHAKOS UNIVERSITY

University Examinations 2017/2018

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR FIRST SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

THIRD YEAR SECOND SEMESTER EXAMINATION FOR BACHELOR OF EDUCATION

SST 201/ SMA 362: OPERATIONS RESEARCH I

DATE: 14/12/2017 **TIME: 2:00 – 4:00 PM**

***Instructions to the Candidate*:**

1. *Answer* ***Question*** **1** *and any other* ***two*** *questions.*
2. *You must have the following items for this paper:*
   - *Scientific Calculator;*
   - *Graph paper.*

1. (a) Outline **four** conditions which must be satisfied for a problem to be solved using linear programming technique. (4 *marks*)

   (b) Explain each of the following terms as used in linear programming:

   (i) Slack variable;

   (ii) Surplus variable;

   (iii) Artificial variable. (6 *marks*)

   (c) A factory produces four types of electrical cables – A, B, C and D. The factory wishes to maximise contribution to revenue. The sale prices per metre of the cables are 40, 32, 60 and 54 Kenya shillings respectively. The factory employs 200 skilled workers and 150 unskilled workers, who work a 40 hour week. The time it takes to produce one metre of each type of cable is as shown in the table below:

Product		A	B	C	D
Hours	Skilled	5	3	2	6
	Casual	3	6	4	8

   Formulate a linear programming model for this problem. (6 *marks*)

(d) Given the linear programming model below:

Minimise : $z = 8x_1 + 12x_2 + 5x_3 + 15x_4$

Subject to : $2x_1 + 3x_2 + 6x_3 + x_4 \geq 20$

$6x_1 + 8x_2 + 5x_3 + 2x_4 \geq 40$

$3x_1 + 5x_2 + 8x_3 + 6x_4 \geq 50$

With : $x_1, \; x_2, \; x_3, \; x_4 \geq 0$

Determine its symmetrical dual program. (2 *marks*)

(e) A lamp production plant produces two products – florescent tubes and energy saving bulbs. Tubes contribute Ksh 60 per unit while bulbs contribute Ksh 80 per unit. The manager of the firm wants to determine the daily production plan which maximises contribution to revenue. The production constraints are as spelt out in the table below:

	Labour hours	Machine hours	Glass material	Coating material
Tubes	3	5	25	1
Bulbs	5	4	18	1
Maximum available	900	1200	4500	200

The Ministry of Energy has issued a directive that a minimum of 40 bulbs must be produced per day in conformity with an energy saving strategy.

(i) Formulate a linear programming model for the problem. (3 *marks*)

(ii) Using the graphical method, determine the optimum production plan which maximises revenue for the firm. (6 *marks*)

(iii) Interpret the optimum solution of the problem obtained in (ii). (3 *marks*)

2. An oil refinery plant produces two oil products – diesel and petrol. The cost of production for diesel is Ksh 40 per litre while the cost of production for petrol is Ksh 50 per litre. The oil products are to be sold in different countries which require both products in specified ratios, but with minimum quantities per month to meet their domestic demand. The oil requirement for the various countries in million litres is as shown in the table below:

Country	A	B	C	D
Diesel	3	8	3	1
Petrol	7	9	2	1
Minimum demand	420	720	240	100

The Ministry of Energy has issued a directive that a minimum of 20 million litres of diesel must be produced per month in conformity with an industrialisation strategy.

(a) Formulate a linear programming model for the problem. (6 *marks*)

(b) Using the graphical method, determine the optimum monthly production plan which minimises the production cost for the refinery. (10 *marks*)

(c) Interpret the optimum solution obtained in (b) above. (4 *marks*)

3. Below is a linear programming model.

Maximise : $z = 5x_1 + 8x_2 + 10x_3$

Subject to : $2x_1 + 3x_2 + x_3 \leq 400$

$x_1 + x_3 \leq 150$

$2x_1 + 4x_3 \leq 200$

$x_2 \leq 50$

With : $x_1, x_2, x_3 \geq 0$

(a) Using the Simplex method, determine the optimum solution for this program. (16 *marks*)

(b) Interpret the solution obtained in (a) above. (4 *marks*)

4. (a) Justify the need for duality approach to solving linear programming problems. (3 *marks*)

(b) Below is a linear programming model. Use it to answer the questions that follow.

Minimise : $z = 180x_1 + 120x_2 + 80x_3$

Subject to : $2x_1 + 2x_2 + x_3 \geq 80$

$3x_1 + x_2 + x_2 \geq 70$

With : $x_1, x_2, x_3 \geq 0$

(i) Determine the symmetrical dual of the LP programming model above. (2 *marks*)

(ii) Using the Simplex method, determine the optimum solution of the dual program. (10 *marks*)

(iii) Using the relationship between a primal program and its symmetrical dual program, derive the optimum solution of the primal program from the optimum solution of its dual. (5 *marks*)

5. A cement factory is required to meet the demands 70, 50, 30, 20 in thousand bags at various destinations from supplies 60, 10, 100 in thousand bags from various sources. The transport cost per unit amount (thousand bags) over the various routes are as given in the matrix below:

$$C = \begin{bmatrix} 2 & 3 & 11 & 7 \\ 1 & 0 & 6 & 1 \\ 5 & 8 & 15 & 9 \end{bmatrix}$$

The company wants to meet the demand at destinations by transporting the oil at the cheapest cost possible. Using the Vogel's approximation method (VAM), for the transportation problem:

(a) Determine the optimal solution that minimizes the cost of transport; (16 *marks*)

(b) Interpret the optimum solution obtained in (a) above. (4 *marks*)