MACHAKOS UNIVERSITY

University Examinations 2019/2020

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS & PROGRAMMING)

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 261: PROBABILITY AND STATISTICS II

DATE: 21/1/2021 **TIME: 8.30-10.30 AM**

INSTRUCTIONS:

Answer question ONE and any other TWO questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Determine the $E(x_1 + x_2)$ from the Bivariate table below. (3 marks)

		x_2	
		0	1
x_1	0	$\frac{1}{6}$	0
	1	⅓	⅓
	2	0	$\frac{1}{6}$

b) If the joint probability density function of X and Y is given by:

$f(x, y) = 3y, 0 \leq y \leq 1, 0 < x < 1$.Determine **F** $(\frac{1}{3}, \frac{1}{2})$ (6 marks)

c) The joint probability density function for x and y is given by $f(x,y) = \frac{1}{21}(x+y)$

$x = 1, 2, 3.$

$y = 1, 2$

i) Determine marginal distribution of x and y (6 marks)

ii) Prove for independence (4 marks)

d) Show that E(X+Y)=E(X)+E(Y) if X and Y are independent continuous random variables (5 marks)

e) Given that $f(x,y) = \begin{cases} 2x, 0 < x, y < 1 \\ 0 \quad otherwise \end{cases}$

Determine $p(0 < x < ⅓, ½ < y < 1)$ (6 marks)

QUESTION TWO (20 MARKS)

a) Given y_1 and y_2 have the joint probability density function (pdf) given by

$$f(y_1, y_2) = \begin{cases} K(1-y_2), & 0 \le y_1 \le y_2 \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

- Determine the value of the constant K. (4 marks)
- Determine $p(y_1 \le \frac{3}{4}, y_2 \ge \frac{1}{2})$ (4 marks)

b) If x and y have joint probability density function $f(x,y) = \frac{8x^2}{7y^3}$ for $1 \le x, y \le 2$

i) Determine the marginal distribution of x and y (8 marks)

ii) Determine the conditional density function of y given $X = x$ (2 marks)

iii) Determine whether x and y are independent. (2 marks)

QUESTION THREE (20 MARKS)

a) Suppose X is a continuous and Y is discrete random variables with a joint p.d.f given by:

$f(x,y) = \frac{yx^{y-1}}{3}$,$0 < x < 1, y \in$ 1,2,3.Determine

(i) $f_y(y)$ (5 marks)

(ii) $E(X)$ (5 marks)

b) Consider the Bivariate table below

	x	y = 0	y = 1
	0	1/8	0
x	1	2/8	1/8
	2	1/8	2/8
	3	0	1/8

(i) Determine *E(x)* (2 marks)
(ii) Determine the covariance of x and y (4 marks)
(iii) The marginal probability mass function of y (4 marks)

QUESTION FOUR (20 MARKS)

a) Let $y_1 \sim B(n_1, p)$ and $y_2 \sim B(n_2, p)$. Determine the moment generating function of $y_1 + y_2$ given that y_1 and y_2 are independent binomial random variables. (6 marks)

b) The joint pdf of x and y is given by that $f(x, y) = \begin{cases} 6x^2 y, & 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$

Determine: (i) $\mu_x \text{ and } \mu_y$ (4 marks)
(ii) $d_x^2 \text{ and } d_y^2$ (4 marks)
(iii) The correlation coefficient between x and y (6 marks)

QUESTION FIVE (20 MARKS)

a) Let y be a continuous random variable with a pdf defined by that

$$f(y) = \begin{cases} 2y, & 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Using the change of variable technique determine the density of $x = 8y^3$. (8 marks)

b) Given that E(y/x)=$\propto + \beta x$, where E denotes expectation, x and y denotes random variables and $\propto$ and β are constants. Show that:

$$\beta = \frac{E(xy) - E(x)E(y)}{E(x^2) - (E(x))^2} = \frac{cov(x,y)}{var(x)}$$ (12 marks)