MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

SECOND YEAR SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (STATISTICS & PROGRAMMING)
BACHELOR OF SCIENCE (MATHEMATICS & COMPUTER SCIENCE)
BACHELOR OF SCIENCE MATHEMATICS
ACHELOR OF EDUCATION
SMA 260: PROBABILITY & STATISTICS 1

DATE: 27/9/2019 **TIME: 8:30 – 10:30 AM**

INSTRUCTION: *Answer Question **ONE** which is compulsory and any other **TWO** Questions*

Question One (30 Marks)

a) Highlight three properties of a normal distribution (3 marks)

b) Suppose the time taken by students to leave a lecture hall is assumed to be normally distributed with mean 10 minutes and standard deviation 4minutes. If a student is randomly taken from the class determine the probability that

i. The student takes exactly 6 minutes

ii. The student takes between 6 to 12 minutes to leave the lecture hall (5 marks)

c) If a random variable X has a pdf

$$f(x)=\begin{cases}60x^3(1-x)^2 & 0<x<1\\ 0 & otherwise\end{cases}$$

Determine the $E(x)$ and $Var(x)$ (4 marks)

d) Given that $F(x)$ is the CDF of a poison distribution with parameter λ and that $F(2)=2F(1)$. Determine the value of λ (5 marks)

e) The probability of winning a certain game is 0.01. if you play the game 10 times determine the probability that you win at most once (3 marks)

f) Suppose a fair coin is tossed 900 times. Determine the probability of head showing up more than 495 times given $E(x)=\frac{1}{2}$ and $Var(x)=\frac{1}{4}$. (5 marks)

g)

h) Suppose the unit rate of receiving calls at an exchange board is 10 calls per hour. Determine in two hour interval;
   i. The expected number of calls and its variance
   ii. The probability of receiving exactly 20 calls (5 marks)

QUESTION TWO (20 MARKS)

a) A pair of fair dice is tossed to obtain equiprobable space Ω consisting of 36 order pairs of numbers between 1 and 6. Determine the $f(x)$ and $F(x)$, represent $F(x)$ in a graph (7 marks)

b) The age of University staff members is normally distributed where 31% of the staff is below 45 years and 8% are over 64 years. Determine the mean and the standard deviation of the age distribution. (5 marks)

c) Suppose X is a random variable with a density function defined as

$$f(x)=\begin{cases}\lambda e^{-\lambda x} & x>0 \quad \lambda>0\\ 0 \quad , & otherwise\end{cases}$$

Determine the mgf_x, $E(x)$ and $Var(x)$ (8 marks)

QUESTION THREE (20 MARKS)

a) Given that X is a random variable with *pdf*

$$f(x)=\begin{cases}\dfrac{1}{2\sqrt{x}} & 1<x<4\\ 0 & elsewhere\end{cases}$$

Determine
   i. The quartile deviation of the random distribution
   ii. The 20^{th} percentile of the random distribution (6 marks)

b) An examination has 10 multiple-choice questions, with 4 choices for each question but only one is correct. To pass a student must answer at least 6 questions correct. If an ill prepared student decided to select the answers randomly determine the probability that the student
   i. Got no answer correct
   ii. Passed the examination (6 marks)

c) If X is the number heads that show up after tossing a fair coin twice, determine the mgf_x, $E(x)$ and $Var(x)$ (8 marks)

QUESTION FOUR (20 MARKS)

a) Given that 20% of the articles taken from a batch of eight are found to be defective. Calculate the probability of getting three or more defective (4 marks)

b) A variable X has a hypergeometric distribution with parameters $A = 8$ and $B = 20$, for what value of n, will $Var(X)$ be maximum (8 marks)

c) The diameter of an electric cable (x) is assumed to be continuous random variable with such that $f(x) = 6x(1-x)$ for $0 < x < 1$ otherwise it is zero.

   i. Confirm that $f(x)$ is a pdf

   ii. Determine a number b such that $p(x < b) = p(x > b)$ (8 marks)

QUESTION FIVE (20 MARKS)

a) A container has 120 alarms of which 5 are defective. If 3 of these alarms are randomly selected and delivered to a customer, determine the probability that the customer will get one defective alarm. (4 marks)

b) i) Highlight four properties of a binomial distribution function (4 marks)

   ii) If X is a binomial random variable show that $Var(x) = npq$ (12 marks)