MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

SMA 203. LINEAR ALGEBRA II

DATE: **TIME:**

INSTRUCTION: *Answer Question **ONE** which is compulsory*

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Give an example to show that every matrix is a zero of its characteristic polynomial (5 marks)

b) Determine whether the transformation $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ defined by

$$T(x_1, x_2, x_3) = (-x_1 + 2x_2 + x_3, \quad x_1 - x_2 + x_3, \quad 2x_1 + x_2 - x_3)$$

is linear or not (4 marks)

c) Calculate $\begin{vmatrix} -1 & 0 & -2 & 3 \\ 1 & 5 & -5 & 0 \\ 2 & -3 & 4 & -4 \\ 1 & 4 & -2 & 0 \end{vmatrix}$ (5 marks)

d) Find a basis or bases for the Eigen space(s) associated with the matrix $E = \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix}$. Determine whether E is diagonalizable or not. (6 marks)

e) Find the nullity and rank of $f: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ if $f(x, y, z) = (x + y - 3z, -x - 2y + 2z, x - 2y + z)$ (4 marks)

f) Determine the kernel and image of T for $T: \Re^3 \to \Re^3$ defined as $T(x, y, z) = (2x - y - z,\ 3x - 2y,\ y - 4z)$ (6 marks)

QUESTION TWO (20 MARKS)

a) Let $T: \Re^2 \to \Re^2$ be a linear transformation defined by $T(x, y) = (2x + 3y,\ 4x - 5y)$ and $S = \{(1,2), (3,5)\}$ and $S' = \{(1,-1), (1,-2)\}$ are bases of $\Re^2$.

Find the

i) matrix representation of T relative to the bases S and S' (6 marks)
ii) change of basis matrix P from S to S' (4 marks)
iii) change of basis matrix Q from S' to S (4 marks)

b) Explain what it means by a diagonal matrix and give an example. (2 marks)

c) Find the minimal polynomial of $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (4 marks)

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QUESTION THREE (20 MARKS)

Consider the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

a) Determine the eigenvalues and bases for the corresponding Eigen spaces of A. (8 marks)

b) Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$. (9 marks)

c) Use the diagonal matrix D in b) above to compute A^7. (3 marks)

QUESTION FOUR (20 MARKS)

a) List down any four properties of determinants. (4 marks)

b) If the matrix $A = \begin{bmatrix} 1 & 3 & 5 \\ 0 & -1 & 3 \\ 2 & 1 & 9 \end{bmatrix}$,

i) state the Sarrus' rule of finding determinants of matrices (2 marks)

ii) determine the determinant of A^{-1} (3 marks)

c) Given matrices $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$, determine if A and B are similar.

(4 marks)

d) Use the Cayley-Hamilton theorem to find the inverse of $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ (7 marks)

QUESTION FIVE (20 MARKS)

a) Find the eigen values and the associated eigen vectors for $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (6 marks)

b) Diagonalize the matrix $A = \begin{pmatrix} 6 & -2 & -1 \\ -2 & 6 & -1 \\ -1 & -1 & 5 \end{pmatrix}$ (8 marks)

c) Let $T : \mathfrak{R}^3 \to \mathfrak{R}^2$ be a linear transformation such that $T(\mathbf{u}) = (1,0), T(\mathbf{v}) = (2,-1)$ and $T(\mathbf{w}) = (4,3)$ where $\mathbf{u} = (1,1,1)$, $\mathbf{v} = (1,1,0)$ and $\mathbf{w} = (1,0,0)$. Find $T(2,-3,5)$ (6 marks)