MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCEINCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

SMA 400: TOPOLOGY

DATE: 24/7/2017 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS

Answer ALL the questions in Section A and ANY TWO Questions in Section B

SECTION A (COMPULSORY)

QUESTION ONE (30 MARKS)

a) Give the definition of a topological space. (4 marks)

b) Let X be a non void set ρ_d be a discrete topology in X. If $E \subset X$ find E^d (the set of all limit points of E). (4 marks)

c) Let $X = \{1,2,3,4\}$. Let $S = \{\{1\},\{2,3\},\{3,4\}\}$ be a collection of subsets of X. Give the topology generated by the set S. (4 marks)

d) Let X be a non-empty set. Define the following topologies

i Trivial topology

ii Discrete topology (2 marks)

e) Let X be a trivial topological space. Find A' the derived set of A. (2 marks)

f) i Define the neighborhood of a point $p \in X$ where X is a topological space.

(2 marks)

g) Let $X = \{a, b, c, d, e\}$ and $\rho = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{c\}, \{abc\}, \{a, c\}, \{b, c\}\}$.

Let $E = \{a, b, c\}$. Find:

i The closure of E

ii The interior of E (4 marks)

h) Find the topology generated in R by the class of all closed sub-intervals of the type $[a, a+1]$. (4 marks)

SECTION B: ANSWER ANY OTHER TWO QUESTIONS

QUESTION TWO (20 MARKS)

a) Find the topology generated by the sub-basis $\beta = \{\{1\}, \{1, 3, 4\}, \{2, 3\}, \{3\}\}$, where $X = \{1, 2, 3, 4\}$ (5 marks)

b) Given any collection of subsets of a nonvoid set X. Will this collection serve as a base for the topology? (5 marks)

c) If N_1 *and* N_2 are two neighborhoods of x, show that $N_1 \cap N_2$ is also a neighborhood of x. (5 marks)

d) Consider R with the usual topology and let $Y = (0,1) \cup \{2\}$ if $E = (\frac{1}{2}, 1)$ find the closure of E in the subspace Y. (5 marks)

QUESTION THREE (20 MARKS)

a) Consider the topology $\tau = \{X, \emptyset, \{1\}, \{3,4\}, \{1,3,4\}, \{1,3,4,5\}\}$ on $X = \{1,2,3,4,5\}$ and the subset

$A = \{b, d, e\}$ of X. Compute showing working out:

$Int(A)$ (3 marks)

$Ext(A)$ (3 marks)

$B'dary(A)$ (4 marks)

b) Let (X, τ) be a topological space. Define a base and sub-base for the topology τ on X.

(2 marks)

c) Let $X = \{1,2,3\}$ and $\tau = \{\{1,2\}, \{2,3\}, \emptyset, \{2\}, \{1,2,3\}\}$. Give the base and sub-base for the topology τ on X. (4 marks)

d) Given a subset A of a topological space, define what it means for p to be a limit point of A. (2 marks)

e) Let X be a topological space and $A \subseteq X$. Give the definition of the closure of A. (2 marks)

QUESTION FOUR (20 MARKS)

a) Let (X, ∂) be a metric space and A $\subset X$. Then prove that if P is a limit point of A every neighborhood of P contains infinitely many points distinct from P. (6 marks)

b) Give the definition of a homeomorphism between two topological spaces X and Y. (3 marks)

c) Let (X, ρ) and (Y, ρ^*) be topological spaces and $f: X \to Y$ be a function if $p \in X$ and$\{p\} \in \rho$ Show that f is continuous. (6 marks)

d) Let (X, ρ) be a topological space show that; If $x \in E^d$ then $x \in (E - \{x\})^d$. (5 marks)

QUESTION FIVE (20 MARKS)

a) Let $X = \{a, b, c, d, e\}$ and $J = \big\{\emptyset, x\{a\}\{b, c\}\{b, c, d\}\{a, b, c\}\{a, d\}\{d\}\{a, b, c, d\}\big\}$ let

$Y = \{a, b, c, d\}$,

i find the induced topology on Y

ii Define relative topology

iii Find the closed set in relative topology (6 marks)

b) Define hereditary as used in topological spaces (2 marks)

c) $T_{0\ and}\ T_1$ spaces are hereditary prove (4 marks)

d) Let (X, ρ) be a topological space (X, ρ) is a T_1 space iff each singleton subset $\{x\}$ is closed in (X, ρ) (4 marks)

e) Consider the discrete topology τ on $X = \{a, b, c, d\}$. Find a sub-base for τ which does not contain any singleton sets. (4 marks)