MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR BACHELOR OF EDUCATION

SPH 400: CLASSICAL MECHANICS

DATE: 31/7/2017 **TIME: 11:00 – 1:00 PM**

INSTRUCTIONS:

- The paper consists of **two** sections.
- Section **A** is **compulsory** (30 marks).
- Answer any **two** questions from section **B** (each 20 marks).

Required data and constants:

Earth's gravitational pull = 9.8 N/Kg

The Universal Gravitational Constant $G = 6.67 \times 10^{-11} N.m^2/Kg^2$

SECTION A (COMPULSORY)

QUESTION ONE (30 MARKS)

a) Define the term virtual displacement (2 marks)

b) i With examples, state the constraints involved in solving Newtonian mechanics problems (3 marks)

ii How are those constraints eliminated (2 marks)

c) i Discuss why is not necessary for generalized coordinates to have dimensions of length. (2 marks)

(ii) State the mathematical conditions that must be satisfied by Generalized coordinates (2 marks)

d) (i) Find the number of degrees of freedom for a rigid body which has one point fixed but can move in space about this point (2 marks)

(ii) A system of particles consists of particles of mass 3g located at point P (1, 0, -1), 5g at point Q (-2, 1, 3) and 2g at point R (3, -1, +1). Find the coordinates of the center of mass of the system (3 marks)

e) State three application of Lagrange's equation for motion of various mechanical system (3 marks)

f) Distinguish the following terms:

(i) Discrete and continuous systems (2 marks)

(ii) Constraints and Degrees of freedom (2 marks)

(iii) Rigid and elastic bodies (2 marks)

g) The escape velocity of a particle on the earth is the minimum velocity required at the surface of the earth in order that the particle can escape from the earth's gravitational field. Neglecting the resistance of the atmosphere, the system is conservative. From conservative theorem for potential and kinetic energy, calculate the escape velocity in ms^{-1} for the earth ignoring the presence of the moon (5 marks)

QUESTION TWO (20 MARKS)

a) Given the sum of external and internal forces acting on k^{th} particles given by:

$M_k\ddot{\boldsymbol{r}}_k = \boldsymbol{F}_k\mathrm{e} + \boldsymbol{F}_k\mathrm{i}$

where $\boldsymbol{F}_k\mathrm{e}$ and $\boldsymbol{F}_k\mathrm{i}$ are external and internal forces respectively.

(i) Use the equation to derive the momentum theory (4 marks)

(ii) Using Newton's third law, prove that $\boldsymbol{F}_k\mathrm{i} = 0$ (3 marks)

(iii) Similarly, using principle of virtual displacement, show that $\boldsymbol{F}_k\mathrm{i} = 0$ (3 marks)

b) Use the Hamiltonian to find the equations of motion of a simple pendulum (10 marks)

QUESTION THREE (20 MARKS)

a) A particle of mass m is attached to spring which passes through a hole in a flat horizontal table. The spring is then fastened to a spring with force constant k. When the particle is at the hole in the table, the spring is un-stretched. Sketch the system, and using plane polar coordinates, apply the Lagrange's equations to find the equations of motion. (10 marks)

b) Find the velocity of a rocket moving vertically upward at any time t in a constant gravitational field. Assume that the gas is ejected at a constant rate and at a constant velocity with respect to the rocket. (10 marks)

QUESTION FOUR (20 MARKS)

a) Using Lagrange's equations, for a system of single particles, with coordinate $\boldsymbol{X}_i$ (where i= 1, 2, 3). Show the eguivalence of Lagranges's and Newton' equations. (6 marks)

b) Find the equation of motion in a one dimensional harmonic oscillator using Hamilton's principle. (7 marks)

c) Deduce the Lagrangian for a particle moving in the x-y plane with components of its position vectors being

$x = a(\theta - \sin\theta)$,

$y = a(\theta + \cos\theta)$, $0 \leq \theta \leq 2\Pi$ (7 marks)

QUESTION FIVE (20 MARKS)

a) A particle of mass m moves on a plane in the field of force given by:

$F = -\boldsymbol{r}krcos\phi$

where k is constant and **r** is the radial unit vector.

(i) Will the angular momentum of the particle about the origin be conserved? Explain your statement (3 marks)

(ii) Obtain the differential equation of the orbit of the particle (3 marks)

b) A body of mass 50g falls from a height 30 cm onto a horizontal surface and re-bounds to height of 20 cm. If time of collision is 0.05 sec. Calculate:

(i) Average force experienced by body during impact (3 marks)

(ii) Kinetic energy of body on impact (2 marks)

(iii) Coefficient of Restitution R between the 2 surfaces (2 marks)

c) Lagrangian equations of motions are applied in determining equations of motion of various mechanical systems.

(i) Deduce the expression for Lagrangian for simple pendulum. (2 marks)

(ii) Use it to obtain Lagrange's equation of motion from the energy point of view. (2 marks)

(iii) Show that it is a similar expression for Newtonian equations. (3 marks)