



MACHAKOS UNIVERSITY

ISO 9001:2008 Certified 

UNIVERSITY SUPPLEMENTARY EXAMINATIONS

SMA 407: MEASURE THEORY

(a) Answer ALL the questions in Section A and ANY TWO Questions in Section B

SECTION A

QUESTION ONE 30 MARKS

- a) Show that if E is Lebesgue measurable subset of $\mathbb{R}$ then E^c is Lebesgue measurable subset (5 marks)
- b) Prove that if μ is a measure defined on a σ -algebra $\mathfrak{X}$. Then μ is monotonic, that is if $T \subset S$, then $\mu(T) \leq \mu(S)$. Furthermore if $\mu(T) < \infty$, then $\mu(S - T) = \mu(S) - \mu(T)$. (5 marks)
- c) Show that if $\mu^*(F) = 0$, then F is μ -measurable. (5 marks)
- d) Prove that the space $(\mathbb{R}, \mathcal{M}, \mu)$, where μ is the Lebesgue measure is complete. (5 marks)
- e) Prove that if $f, g: X \rightarrow \mathbb{R}$ are two $\mathfrak{X}$ -measurable functions and c be a real number then the function cf is $\mathfrak{X}$ -measurable (5 marks)
- f) Let (f_n) be a sequence of $\mathfrak{X}$ -measurable function $f_n: X \rightarrow \mathbb{R}$. Show that the set of points over which (f_n) converges point wise is measurable. (5 marks)

SECTION B

QUESTION TWO 20 MARKS

- a) Prove that if $f, g: \mathfrak{X} \rightarrow \mathbb{R}$ are two $\mathfrak{X}$ –measurable functions and c be a real number then the function
- $c + f$
 - fg are $\mathfrak{X}$ –measurable. (@5 marks)
- b) Let (f_n) be a sequence of $\mathfrak{X}$ –measurable functions $f_n: X \rightarrow \mathbb{R}$, then the functions $f_1 \cup f_2 \cup f_3 \cup \dots \cup f_n$ and $f_1 \cap f_2 \cap f_3 \cap \dots \cap f_n$ is $\mathfrak{X}$ –measurable. (@5 Mrk)

QUESTION THREE 20 MARKS

- Prove that $\mu^*(\{y\}) = 0$ for all $x \in \mathbb{R}$ (4 marks)
- Prove that if $A \subset B$ then $\mu^*(A) \leq \mu^*(B)$, $A, B \in \mathfrak{R}$ (5 marks)
- Let $A = \{x \in \mathbb{R}: -3 < x < 4\} \cup \{x \in \mathbb{R}: 1 \leq x \leq 6\}$. Find $\mu^*(A)$ (4 marks)
- Prove that lebesgue outer measure μ^* is countably sub additive. (7 marks)

QUESTION FOUR 20 MARKS

- Prove that if f and g both belong to M^+ and $f \leq g$, then $\int f du \leq \int g du$ (6 marks)
- Prove that if $f \in M^+$ and if $E, F \in \mathfrak{X}$, with $E \subset F$ then $\int_E f du \leq \int_F f du$ (7 marks)
- State and prove Fatous lemma. (7 marks)

QUESTION FIVE 20 MARKS

- State and prove monotone convergence theorem. (10 marks)
- Let $(X, \mathfrak{X})$ be a measurable space. Then if $f, g \in M^+(X, \mathfrak{X})$, then $f + g \in M^+(X, \mathfrak{X})$ (6 marks)
- Prove that if φ and ρ are simple functions in $M^+(X, \mathfrak{X})$ and $c \geq 0$, then $\int c\varphi du = c \int \varphi du$ (4 marks)