



# MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

Course

Second Year Second Semester

ECU 202: ENGINEERING MATHEMATICS VI

Date: .....

Time: .....

*Answer question ONE and any other TWO questions*

## **QUESTION 1: 30 MARKS (COMPULSORY)**

- a) Define
  - i. Differential equations
  - ii. Order of a differential equation
  - iii. Ordinary differential equation (3 marks)
- b) Show that the function  $y = \cos x$  is a solution of the ordinary differential equation defined by
$$\frac{d^2y}{dx^2} + y = 0$$
(3 marks)
- c) Form a differential equation from the primitive defined by  $y = Ax^2 + Bx$  (5 marks)
- d) Solve the differential equation defined by  $\frac{dy}{dx} = \frac{x^2-2}{y}$  (5 marks)
- e) Find the orthogonal trajectories of the curves whose equation is  $x^2 - y^2 = c$  (5 marks)
- f) Solve the differential equation defined by  $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$  (5 marks)
- g) Define the particular solution of  $\frac{d^2y}{dx^2} + 4y = \cos 3x$  (4 marks)

## **QUESTION 2 (20 MARKS)**

- a) Find the orthogonal trajectories of a curve whose equation is  $y = \frac{x}{c+x}$  (5 marks)
- b) Form a differential equation from the primitive defined by  $y = A \cos 2x + B \sin 2x$

(5 marks)

- c) A substance cools from  $100^{\circ}\text{C}$  to  $60^{\circ}\text{C}$  in 10 seconds. Find the temperature of the substance after 40 seconds assuming that the room temperature is  $20^{\circ}\text{C}$ . Given that  $\frac{dT}{dt} \propto (T - T_0)$ . With  $T$  being the temperature at time  $t$  and  $T_0$  being the room temperature (10 marks)

### **QUESTION 3 (20 MARKS)**

- a) Find the particular solution of the ordinary differential equation given by

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = x - 2 \quad (6 \text{ marks})$$

- b) Find the solution of the differential equation defined by  $\sin x \cos y \, dx + \cos x \sin y \, dy = 0$  (4 marks)

- c) Using the method of undetermined coefficients solve the differential equation defined by

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 4x^2 \quad (10 \text{ marks})$$

### **QUESTION 4 (20 MARKS)**

- a) solve the linear differential equation defined by

$$\frac{dy}{dx} - \frac{y}{x} = x^2 \quad (6 \text{ marks})$$

- b) Show that the solution of the differential equation defined by

$$\cos y \sin x \, dx + \sin y \cos x \, dy = 0 \text{ is } \cos x \cos y = A \quad (7 \text{ marks})$$

- c) Use Laplace transform to solve the differential equation defined by

$$\frac{dy}{dx} - 5y = 0 ; y(0) = 2 \quad (7 \text{ marks})$$

### **QUESTIONS 5 (20 MARKS)**

- a) Determine whether the differential equation below is exact and find the solution of the differential equation

$$2xy \, dx + (1 + x^2) \, dy = 0 \quad (7 \text{ marks})$$

- b) Show that the solution of an homogeneous linear differential equation with roots that are real and equal is given by

$$y = (Ax + B)e^{\alpha x} \quad (4 \text{ marks})$$

- c) Solve the following differential equation

$$x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} - \frac{2}{x}y = \ln x \quad (9 \text{ marks})$$