

MACHAKOS UNIVERSITY

University Supplementary Examinations

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

ECU 301: ENGINEERING MATHEMATICS X

Date:

Time:

Instructions to Candidates

Answer question ONE and any other TWO questions

QUESTION 1: 30 MARKS (COMPULSORY)

a) Prove each of the following

i) $\nabla \Delta = \Delta - \nabla$ (4 marks)

ii) $\mu^2 f_i = \frac{1}{4}(f_{i+1} + 2f_i + f_{i-1})$ (3 marks)

b) Derive Newton's Forward Difference Interpolating Polynomial of degree n (4 marks)

c) Evaluate the integral below using Trapezoidal rule with 4 and 8 subdivisions

$\int_0^2 \frac{1}{1+x} dx$ (6 marks)

d) Show that the relative error in a product is equal to the the sum of the relative errors

$(V_{xy} = V_x + V_y)$ (3 marks)

e) Using Lagrange's quadratic interpolating polynomial find the value of f(x) corresponding to x=2.3

x	1.1	1.7	3.0
f(x)	10.6	15.2	20.3

(5 marks)

f) Use linear interpolation to estimate the value of velocity at t=2.6 given the data below

t (s)	1	2	3
v(t) (m/s)	2.03	4.23	6.84

(5 marks)

QUESTION 2 (20 MARKS)

a) Evaluate $\int_0^1 \frac{1}{1+x} dx$ using Gaussian three point integration rule. Find the exact value and the error in the calculation in 6 d.p (10 marks)

b) Given the data

x	-4	-3	-2	-1	0	1	2	3	4
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f(x)	-3	-65	-49	-15	1	-13	-45	-69	5
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- i) Locate the error in the data given and obtain a corrected finite difference table (6 marks)
- ii) Extend the table to obtain the value of $f(5)$ (4 marks)

QUESTION 3 (20 MARKS)

- a) Use Newton-Raphson iterative formula to get a root of the equation given below

$$x^3 - 3x + 1 = 0 \text{ Near } x=2 \text{ to 4 d.p} \quad (8 \text{ marks})$$

- b) The profits of a company are as given in the table below (in thousands)

Year (x)	2000	2003	2006	2009	2012
Profit $y=f(x)$	120	100	111	108	99

- i) Calculate the profits for the year 2001 ($f(2001)$) using Newton Forward difference polynomial (6 marks)
- ii) Calculate the rate of change of profit for the year 2001 ($f'(2001)$) using Newton Forward difference polynomial (6 marks)

QUESTION 4 (20 MARKS)

- a) Derive the linear interpolating polynomial (4 marks)
- b) Use the above (in a) derivation to estimate the value of $f(2.05)$ give the data in the table below;

x	2.0	2.1	2.2	2.3	2.4
f(x)	2.218	2.628	3.058	3.508	3.978

(6 marks)

- c) Use Taylor series method to find the solution of the initial value problem given as

$$\frac{dy}{dt} = t + y; y(1)=0 \text{ at } t=1.2, h=0.1 \quad (10 \text{ marks})$$

QUESTIONS 5 (20 MARKS)

- a) Given the initial value problem below, find y approximately at $x=0.1$ in 5 steps using Euler's method

$$\frac{dy}{dt} = \frac{y-t}{y+t}; y(0)=1 \quad (8 \text{ marks})$$

- b) Solve the following differential equation using 4th order Runge-Kutta method from $t=0$ to $t=0.2$ taking $h=0.1$ (12 marks)