



MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE IN ANALYTICAL CHEMISTRY

SMA 262: DIFFERENTIAL EQUATIONS

DATE: 6/12/2021

TIME: 8.30-10.30 AM

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Give the order of the given differential equation

i. $xy\left(\frac{d^2y}{dx^2}\right)^3 - y^2 \sin x = 0$ (2 marks)

ii. $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ (2 marks)

- b) Solve $x\frac{dy}{dx} = 5x^4 + 4x^2 + 2x$ by direct integration (4 marks)

- c) Find the general solution to $(D^3 - 3D + 2)y = 0$ (4 marks)

- d) Find the general and singular solution of $y = px + \frac{a}{p}$ (4 marks)

- e) Test for exactness and hence solve the following differential equation

f) $(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$ (4 marks)

- g) Reduce the given differential equation to homogenous form and solve it

$(2x + 3y - 5)\frac{dy}{dx} + (3x + 2y - 5) = 0$ (10 marks)

QUESTION TWO (20 MARKS)

a) Form the differential equation associated with $ay^2 = x^2$ (3 marks)

b) Use the method of undetermined coefficient to solve

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 4x^2 \quad (7 \text{ marks})$$

c) Find the orthogonal trajectories of the family $x^2 + 3y^2 = cy$. Find that member of the orthogonal trajectories which passes through the point (1,2). (10 marks)

QUESTION THREE (20 MARKS)

a) Find the general solution corresponding to the equation $(D^2 + 4)y = \sin^2 x$ (6 marks)

b) Find a suitable integrating factor and solve the equation

$$xydx - (x^2 + 2y^2)dy = 0 \quad (7 \text{ marks})$$

c) Apply the method of separation of variables to solve

$$\frac{dy}{dx} + xy = xy^3 \quad (7 \text{ marks})$$

QUESTION FOUR (20 MARKS)

a) Verify that $Ax^2 + By^2 = 1$ is the solution to $xy \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx} \right)^2 - y \frac{dy}{dx} = 0$ (10 marks)

b) If the population of a country doubles in 50 years. In how many years will it triple under the assumption that the rate of increase is proportional to the number of inhabitants? (10 marks)

QUESTION FIVE (20 MARKS)

a) Find the differential equation of the family $y = Ae^{2x} + Be^{-2x}$ (4 marks)

b) Define and solve the given first order differential equation.

$$x^2(x^2 - 1) \frac{dy}{dx} + x(x^2 + 1)y = x^2 - 1 \quad (16 \text{ marks})$$