MACHAKOS UNIVERSITY

University Examinations for 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF EDUCATION (SPECIAL NEEDS)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

SMA 330: NUMERICAL ANALYSIS I

DATE: 9/8/2021 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS:

Attempt question ***one (compulsory***) and any other ***two questions***.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Given that $f(x_1\ x_2\ x_3) = \dfrac{5x_1x_2^2}{x_3^3}$, and that Δx_1, Δx_2, *and* Δx_3 denote the errors in x_1, x_2 *and* x_3 respectively such that $x_1 = x_2 = x_3 = 1$ and $\Delta x_1 = \Delta x_2 = \Delta x_3 = 0.001$ Determine the relative maximum error in evaluating f (4 marks)

b) Determine the unique polynomial $p(x)$ of degree 2 or less such that $p(1) = 1, \quad p(3) = 27, \quad p(4) = 64$ using Newton's divided difference interpolation formula

(4 marks)

c) Derive the n^{th} order numerical relationship between the forward finite difference Δ, the backward finite difference ∇, and the central finite difference δ at x_i (4 marks)

d) A real root of the equation $e^x \sin x = 1$ lies in the interval $(0.5, 0.6)$. Perform two iterations of the Regular-Falsi method to determine this root. (4 marks)

e) Consider the table below

x	6.0	6.1	6.2	6.3	6.4
$f(x)$	0.1750	-0.1998	-0.2223	-0.2422	-0.2596

Determine $f''(6.3)$, $error = O(h^2)$ (4 marks)

f) Estimate the maximum error in $I = \int_0^1 5x^4 dx$ using Simpson's rule with $n = 4$ (5 marks)

g) Show that $\left[1 + \frac{\delta^2}{4}\right]^{\frac{1}{2}} = \frac{1}{2}\delta$ (5 marks)

QUESTION TWO (20 MARKS)

Consider the polynomial $f(x) = (1-x)^{\frac{1}{2}}$, $x \in [0,\ 1]$

a) Obtain a second degree polynomial using Taylors series expansion about $x = 0$. (10 marks)

b) Use the expansion to approximate (0.1) (6 marks)

c) Calculate a bound of the truncation error for $x \in [0,\ 1]$ (4 marks)

QUESTION THREE (20 MARKS)

Consider the table below.

x	0	1	2	3	4
y_x	1	1.5	2.2	3.1	4.6

a) Construct a forward difference table for the data (4 marks)

b) Estimate;

i.) $\Delta^3 y_1$ (5 marks)

ii.) y_x (6 marks)

iii.) y_6 (5 marks)

QUESTION FOUR (20 MARKS)

a) Using Lagrange's formula. Determine a polynomial the the data

x	0	1	3	4
y	-12	0	6	12

(10 marks)

b) Determine a real root of the equation $\sin x = 1 + x^3$ between -2 *and* -1 correct to 3 *dp* by Newton-Raphson method.

(10 marks)

QUESTION FIVE (20 MARKS)

a) A particle moves along a straight line so that at time t seconds its distance s from a fixed point is given by;

$$\frac{ds}{dt} = t\left(\sqrt{8 - t^3}\right)$$

Using Simpson's Rule with 8 *strips* calculate the approximate distance travelled by the particle from $t = 0$ *to* $t = 2$ (8 marks)

b) Consider the table below;

x	1.2	1.3	1.4	1.5	1.6
y	0.9320	0.9636	0.9855	0.9975	0.996

Determine;

i.) The value of x *to* 2*dp* for which y is maximum (7 marks)

ii.) This maximum value of y (5 marks)