MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

FOUTH YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION IN SCIENCE & ARTS.

SMA433-PARTIAL DIFFERENTIAL EQUATIONS II

DATE:………………………………… TIME:…………

INSTRUCTION TO CANDIDATES: ANSWER **QUESTION ONE** AND **ANY TWO** OTHER QUESTIONS.

Question 1 (Compulsory) (30marks)

a) Evaluate the solution to the following initial value problem

$$u_{xx} = 4xy + e^x$$

with initial conditions

$$u(0, y) = y,\ u_x(0, y) = 1$$ (5 marks)

b) Solve the equation $\dfrac{\partial^4 z}{\partial x^4} + \dfrac{\partial^4 z}{\partial y^4} = 2\dfrac{\partial^4 z}{\partial x^2 \partial y^2}$ (5 marks)

c) Find the particular integral of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial z}{\partial y} = 4e^{2x+4y}$$ (6 marks)

d) Consider the equation

$u_{xx} + 4u_{xy} + 4u_{yy} = 0$. Classify the equation and find the general solution. (6 marks)

e) A circular cylinder of radius a has its surface kept at a constant temperature $\theta_{0.}$ if the internal temperature is zero throughout the cylinder, prove that for $t > 0$,

$$\theta(r,1) = \theta_0 \left\{ 1 - \frac{2}{\omega} \sum_{n=1}^{\infty} \frac{J_0(\xi_n r)}{\xi_n J_n(\xi_n a)} e^{-\xi_n^2 kt} \right\}$$

Where $\pm\xi_0, \pm\xi_1, \ldots\ldots \pm\xi_n$ are roots of $J_0(\xi_n) = 0$ (8 marks)

Question 2 (20 Marks)

a) Obtain a partial differential equation of the first order by eliminating the arbitrary function

$$u = e^{-y} f(x) + e^x g(y)$$ (4 marks)

b) Reduce the equation $r - x^2 t = 0$ to canonical form (8 marks)

c) Show that Laplace's equation in three dimensions

$u_{xx} + u_{yy} + u_{zz} = 0$ (8 marks)

Question 3 (20 Marks)

a) i) Find the characteristics of equation $r - yt = 0$

ii) Show under which condition the curve will be a characteristic of the partial differential equation $p^2 r + 3pqs + 2q^2 t = 0$ (5 marks)

b) Find the complete solution of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = (x - y)$$ (5 marks)

c) Reduce the equation $r + 2s + t = 0$ into canonical form and solve completely the corresponding equation. (10 marks)

Question 4(20 marks)

a) If i.) $(\alpha_r D_1 + \beta_r D_2 + \wp_r)$, is a linear factor of the linear differential operator $F(D_1 D_2)$

ii.) $\phi_r(\xi)$, is arbitrary function of one variable ξ

iii.) $\alpha_r \neq 0$

Prove that $z_r = \sum_{r=1}^{n} \exp\left(-\frac{\wp_r}{\alpha_r} x\right) \phi_r(\beta_r x - \alpha_r y)$ is the solution of the reduced equation $F(D_1 D_2) z = 0$ (10 marks)

b) Find the general solution of the partial differential equation

$x^2 z_{xx} - 2xyz_{xy} - y^2 z_{yy} - xz_x + yz_y - 5z = 7x - 5y$ (10 marks)

Question 5 (20 Marks)

Given the partial differential equation

$r - t = 0 \ldots\ldots$ (i)

and five real functions $\left.\begin{array}{r} x = t \\ y = 0 \\ z = t^2 + t^4 \end{array}\right\} \ldots\ldots$(ii) and

$\left.\begin{array}{r} p = 2t + 4t^3 \\ q = 4t \end{array}\right\} \ldots\ldots$(iii).

Show that the five real fuctions form a strip hence interpret geometrically initial value problem in which we seek a solution of the form $z = z(x, y)$ (20 marks)