



MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS

BACHELOR OF EDUCATION

SMA 230: VECTOR ANALYSIS

DATE:

TIME:

INSTRUCTIONS

Attempt question *one (compulsory)* and any other *two questions*.

QUESTION ONE (30 MARKS) (COMPULSORY)

- a) Consider the region bounded by the equations $y = \sin x$ and $y = 0$ from the line $x = 0$ to $x = \pi$. Calculate the centroid (center of mass) of the region (3 marks)
- b) Calculate the area bounded by the curve $y = \frac{1}{\sqrt{x}}$, x -axis and the line $x = 1$ (3 marks)
- c) Evaluate $\int \left(\frac{2x^3 + 5x^2 + 8x + 9}{x} \right) dx$ (4 marks)
- d) Find the length of the arc from $\theta = 0$ to $\theta = \frac{\pi}{4}$ of the curve given by $x = 3\cos \theta$, $y = 3\sin \theta$.
- e) Evaluate the definite integral $\int_0^2 \cos^2 x dx$ (5 marks)
- f) Sketch the curve $y = 2x^2$ from $x = 0$ to $x = 5$. Hence evaluate the area enclosed by the curve, x -axis and the ordinates $x = 1$ to $x = 4$ (5 marks)
- g) Find the area between the curves $x^2 = y$ and the straight line $y = 2$ (5 marks)

- h) Show that the improper integral $\int_1^{\infty} \frac{1}{x^2} dx$ is convergent (5 marks)

QUESTION TWO (20 MARKS)

- a) Evaluate the improper integral $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$ (3 marks)

- b) Evaluate by adjustment of the derivative $\int \frac{x^2 + x + 1}{2x^3 + 3x^2 + 6x + 5} dx$ (4 marks)

- c) Use appropriate technique to evaluate the integral

$$\int \sin^4 x dx \quad (6 \text{ marks})$$

- d) Use partial fractions to evaluate the integral

$$\int \frac{3x^2 - 2x - 7}{x^2 - x - 2} dx \quad (7 \text{ marks})$$

QUESTION THREE (20 MARKS)

- a) Prove that $\int e^{ax+b} dx = \frac{e^{ax+b}}{a} + c$ where a and b are constants (4 marks)

- b) Use the fact that $\sin^2 x + \cos^2 x = 1$ to evaluate the integral

$$\int \cos^3 x \sin^2 x dx \quad (4 \text{ marks})$$

- c) A particle carrying a positive electrical charge $+q$ is released at a distance d from an immovable positive charge $+Q$. Calculate amount of work done on the particle as its distance increases to $2d$. (4 marks)

- d) Find the surface area generated by the loop of the curve $x = t^2, y = t - \frac{t^3}{3}$ about the axis.

(8 marks)

QUESTION FOUR (20 MARKS)

- a) Use integration by parts to evaluate

i. $\int x^3 \ln x dx$ (5 marks)

ii. $\int x^2 e^{2x} dx$ (5 marks)

b) Draw the curve $6y^2 = x(x-2)^2$ and calculate the length of the loop of the curve

$$y^2 = \frac{1}{6}x(x-2)^2 \quad (5 \text{ marks})$$

c) Find the volume obtained by rotating the area under the curve $y = 1 + x$ between $x = 1$ and $x = 2$ about the axis of x . (5 marks)

QUESTION FIVE (20 MARKS)

a) Evaluate;

i.) $\int_0^{\pi} x \sin^6 x \cos x^4 x dx$ (3 marks)

ii.) $\int_0^{\frac{\pi}{4}} \log(1 + \tan \theta) d\theta$ (6 marks)

b) Compute $\int_0^1 \frac{\tan^{-1} x}{x^{\frac{3}{2}}} dx$ using Simpson's rule with 10 *equal* intervals. (11 marks)