MACHAKOS UNIVERSITY

University Examinations 2017/2018

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEARSECOND SEMESTER EXAMINATIONS FOR BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

SMA 360: PROBABILITY AND STATISTICS IV

DATE: 5/12/2017 **TIME: 8.30-10.30 AM**

INSTRUCTIONS: Answer Questions ONE and any other TWO questions

QUESTION ONE (30 MARKS)

a) Describe two properties of continuous random vector (2 marks)

b) The mgf of a r.v Y is given by $M(t) = \frac{1}{6}e^{t} + \frac{1}{3}e^{2t} + \frac{1}{2}e^{3t}$ Find the mean and variance of Y

(4 marks)

c) Discuss Three properties of joint distribution function (3 marks)

d) Describe three properties of Multivariate Normal distribution (3 marks)

e) Suppose $\mathrm{m}(\tilde{\mathrm{t}}) = [\mathrm{P}_1\mathrm{e}^{\mathrm{t}_1} + \mathrm{P}_2\mathrm{e}^{\mathrm{t}_2} + (1 - \mathrm{P}_1 - \mathrm{P}_2]^{\mathrm{n}}$ obtain $\mathrm{E}[\mathrm{x}_1]$, $\mathrm{E}[\mathrm{x}_2]$, $\mathrm{E}[\mathrm{x}_1\mathrm{x}_2]$ and $\mathrm{E}[\mathrm{x}^2]$

(6 marks)

f) Suppose $\mathrm{X} \sim \mathrm{N}(\mu\ \delta^2)$ find the characteristic function of a standard normal variable $z = \frac{x-\mu}{\delta}$

(6 marks)

g) A random vector $\tilde{X}$ is defined by $\tilde{X} = [X_1, X_2, X_3]$ having 4-variate pdf $f(\tilde{x}) = \begin{cases} 16x_1, x_2, x_3, & 0 < x < 1 \\ 0, & elsewhere \end{cases}$ calculate $p(x\epsilon D) = \{(x_1, x_2, x_3)\ \ x_1 < \frac{1}{2}\ x_2 < \frac{1}{2}$ (6 marks)

QUESTION TWO (20 MARKS)

a) Three random variables having the joint pdf given by $f(x_1, x_2, x_3) = \begin{cases} c(x_1 + 2x_2 + 3x_3, & 0 < x < 1 \\ 0, & elsewhere \end{cases}$

i) Determine constant c (3 marks)

ii) Marginal pdf of x_1 and x_2 (3 marks)

iii) The $P(x_3 < \frac{1}{2} |, x_1 = \frac{1}{4} \ x_2 = \frac{3}{4})$ (4 marks)

b) Let x have a Multivariate Normal distribution with covariance matrix $\sum = \begin{bmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & 0 \\ \rho^2 & 0 & 1 \end{bmatrix}$ show that the conditional distribution of (X_1, X_2) given $X_3 = x_3$ is also MVN with mean $\mu = \begin{bmatrix} \mu_1 + \rho^2(x_3 = \mu_3) \\ \mu_2 \end{bmatrix}$

and covariance matrix $\begin{bmatrix} 1 - \rho^4 & \rho \\ \rho & 1 \end{bmatrix}$ (10 marks)

QUESTION THREE (20 MARKS)

a) Given the random vector $\tilde{X}' = [X_1, X_2, X_3]$ has a covariance $\sum$ given by $\sum = \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix}$ find the mean and variance covariance matrix of the random vector $\tilde{Y}' = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ where $y_1 = x_1 - 2x_2 + 2x_3$, $y_2 = 3x_1 + x_2 - x_3$ (8 marks)

b) Proof that for MVN distribution variables are uncorrelated ⇔ variables are independent (12 marks)

QUESTION FOUR (20 MARKS)

a) Define a characteristic function (2 marks)

b) Given the binomial distribution

$$f(x,y) = \frac{n!\, p_1^x p_2^y (1-p_1-p_2)^{n-x-y}}{x!y!(n-x-y)!} \quad with\ x + y \leq 1\ and\ \sum_{i=1}^{3} p_i = 1$$

i) Find conditional distribution of $y\ given\ X = x$ (14 marks)

ii) $E[Y|X = x] and\ var(Y|X)$ (4 marks)

QUESTION FIVE (20 MARKS)

Suppose $\check{X} = [X_1, X_2, X_3]$ has a three variate normal distribution with density $f(\check{x}) = C\ exp\left(-\frac{1}{2}Q\right)$ where $Q = \frac{1}{17}[11x_1^2 + 7x_2^2 + 5x_3^2 - 6x_1x_2 - 4x_1x_3 - 2x_2x_3 + 2x_1 - 16x_2 - 22x_3 + 48$ identify $\check{\upsilon}$ and Σ hence find c (20 marks)