MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF EDUCATION

SMA 334: FLUID MECHANICS II

DATE: 29/3/2021 **TIME: 11.00-1.00 PM**

INSTRUCTIONS TO CANDIDATES

Attempt question ***one (compulsory)*** and any other ***two questions***.

QUESTION ONE (COMPULSORY – 30 MARKS)

(a) Define the following terms as used in fluid mechanics

(i) A stream line (1 mark)

(ii) A source (1 mark)

(iii) Incompressible fluid (1 mark)

(b) Determine the equation of a straight line passing through the point (a, 0) if the two dimensional flow is described by $u = \frac{-y}{b^2}$ and $v = \frac{x}{a^2}$. (6 marks)

(c) Given that $\phi = 2xy$ determine ψ. (5 marks)

(d) Find the stream function of the two dimensional motion due to the equal sources and an equal sink midway between them. Sketch the streamlines (6 marks)

(e) Find the image of $|z - 2i| = 2$ under the mapping $w = \frac{1}{z}$ (5 marks)

(f) If ϕ and ψ are functions satisfying Laplace equation, show that $S + it$ is analytic where $S = \frac{\partial\phi}{\partial y} - \frac{\partial\psi}{\partial x}$ at $t = \frac{\partial\phi}{\partial x} + \frac{\partial\psi}{\partial y}$. (5 marks)

QUESTION TWO (20 MARKS)

(a) Prove that $\psi = \log[(x-1)^2 + (y-2)^2]$ is harmonic in every region which does not include the point $(1,2)$. Find a function ϕ such that $\phi + i\psi$ is an analytic function of the complex variable $z = x + iy$. Express $\phi + i\psi$ as a function of z. (10 marks)

(b) Show that the following factor represents possible flow phenomena of irrotational type. $\phi = u(x)\frac{\theta}{r}$ (10 marks)

QUESTION THREE (20 MARKS)

(a) A viscous fluid is confined on one side of an infinite porous plane. If the velocity of the suction v_0 at the plane is constant and also the plane moving parallel to itself with constant velocity u, find the velocity distribution and calculate the drag at the plane. (10 marks)

(b) State and derive the Blasius theorem. (10 marks)

QUESTION FOUR (20 MARKS)

(a) (i) State THREE types of standard transformations. (3 marks)

(ii) For the conformal transformation $w = z^{\frac{1}{2}}$, show that the circle $|z-1| = 1$ transforms into $\rho^2 = 2\cos 2\phi$, where $w = \rho e^{i\theta}$ in the w-plane. (7 marks)

(b) Discuss in brief the following

(i) Lubrication theory (5 marks)

(ii) Role of Reynold's number in boundary layer (5 marks)

QUESTION FIVE (20 MARKS)

(a) Derive the Euler's equation

$$\rho\frac{\partial\vec{v}}{\partial t} + \rho\vec{v}.\nabla\vec{v} = -\nabla p$$

Where ρ, $\vec{v}$ and p are density, velocity an pressure respectively. (10 marks)

(b) A fluid is flowing steadily between two fixed parallel plates under a constant pressure gradient $p = \left(\frac{-\partial p}{\partial x}\right)$. Show that the velocity distribution is $u = \frac{p}{2\mu}(d^2 - y^2)$ where 2d is the distance between the plates and y is measured from the mid-point. (10 marks)