



MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS FOR 2016/2017 ACADEMIC YEAR

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF
BACHELOR OF SCIENCE (MATHEMATICS, MATHEMATICS AND COMPUTER
SCIENCE & STATISTICS AND PROGRAMMING)
&
BACHELOR OF EDUCATION (SCIENCE & ARTS)

SMA363: TEST OF HYPOTHESIS I

Date: _____

Time: _____

Instruction: Answer Question One and any other Two Questions

QUESTION ONE

- (a) Differentiate between the following terms
- (i) Simple and composite hypotheses [2 marks]
 - (ii) Type I and type II errors [2 marks]
 - (iii) Most powerful and uniformly most powerful tests [2 marks]
- (b) Let p be the probability that a coin will fall head in a single toss in order to test $H_0: p = \frac{1}{2}$ against $H_1: p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is to reject if more than 3 heads are obtained. Find the size and the power of the test. [6 marks]
- (c) A random sample of 10 students from a normal population showed average of 66kgs and standard deviation of 10kgs. Show whether or not the assumption of an average weight of 70kgs for students in the population is reasonable [5 marks]
- (d) The weight of a new package of fat is known to have a normal distribution with a mean of 40 grams. A random sample of seven packages yielded the following weights:
42, 18, 41, 40, 3, 18, 37
- Test the hypothesis that the standard deviation is less than 4 at $\alpha = 5\%$ [6 marks]

- (e) Consider the following data on the number of hours which 10 persons studied for French test and their scores

Hrs studied X	4	9	10	14	4	7	12	21	1	17
Test score Y	31	58	65	73	37	44	60	91	21	84

Test the hypothesis $H_0: \beta = 3$ against

$$H_1: \beta > 3$$

At $\alpha = 0.01$ level of significance

[7 marks]

QUESTION TWO

- (a) Let $x_1, x_2, \dots, x_n$ be a random sample of size n from random variable X which normally distributed with mean μ and variance σ^2 , where σ^2 are known. Use Neyman Pearson criterion to obtain the best critical region for testing hypothesis $H_0: \mu = \mu_0$ against $H_1: \mu > \mu_0$, μ_0 is specified. [14 marks]
- (b) When a standard lubricant is used on the bearing of a wheel, the length of time for which the wheel will spin in an experiment has a mean of 200s and standard deviation of 10s. A new improved lubricant is tried out and five spins take time in seconds: 209, 224, 196, 219 and 212. Assuming that the standard deviation is unchanged, test whether the new lubricant has significantly increased spinning. Use 5% level of significance [6 marks]

QUESTION THREE

- (a) Let $x_1, x_2, \dots, x_n$ be a random sample of size n from random variable X which normally distributed with mean μ and variance σ^2 , where μ is unknown. Obtain the likelihood ratio critical region for testing hypothesis $H_0: \sigma = \sigma_0$ against $H_1: \sigma \neq \sigma_0$, σ_0 is specified [15 marks]
- (b) In a random sample of the weights of 19 babies of a certain age, the standard deviation was found to be 2.5kgs. Test the hypothesis $H_0: \sigma = 3$ against $H_1: \sigma \neq 3$ at $\alpha = 0.05$ [5 marks]

QUESTION FOUR

- (a) Let $x_1, x_2, \dots, x_m$ be a random sample of size m from random variable X which normally distributed with mean μ_1 and variance σ_1^2 . Also, let $y_1, y_2, \dots, y_n$ be a random sample of

size n from random variable X which normally distributed with mean μ_2 and variance σ_2^2 .

Assuming that $\sigma_1^2 = \sigma_2^2 = \sigma^2$, σ^2 unknown, derive a test statistic for testing

$H_0: \mu_1 = \mu_2$ against $H_1: \mu_1 \neq \mu_2$.

[15 marks]

- (b) A botanist is interested in comparing growth response of dwarf pea stems to two different levels of hormone indoleacetic acid. Let X denote the growth attributed to level I and Y to level II. Eleven observations on X were taken and 13 on Y as follows

X: 0.8, 1.8, 1.0, 0.1, 0.9, 1.7, 1.0, 1.4, 0.9, 1.2, 0.5

Y: 1.0, 0.8, 1.6, 2.6, 1.3, 1.1, 2.4, 1.8, 2.5, 1.4, 1.9, 2.0, 1.2

Assuming $X \sim N(\mu_1, \sigma^2)$ and $Y \sim N(\mu_2, \sigma^2)$, test the hypothesis $H_0: \mu_1 = \mu_2 = \mu$ against

$H_0: \mu_1 \neq \mu_2$ at $\alpha = 0.05$ [5 marks]

QUESTION FIVE

- (a) Consider a simple regression model $Y_i = \alpha + \beta x_i + e_i$, where α and β are constants and $e_i \sim N(0, \sigma^2)$. Derive the test statistic for testing hypothesis $H_0: \beta = 0$ against $H_1: \beta \neq 0$

[15 marks]

- (b) The survival rate Y of a certain breed of pigs was thought to be linearly related to amount X of a drug put in their diet. The model assumed has simple linear and data collected is given below

Y	3	6	7	10	11
X	0	1	2	3	4

Test the hypothesis $H_0: \beta = 2$ against

$H_0: \beta \neq 2$

At $\alpha = 0.05$ level of significance

[5 marks]