MACHAKOS UNIVERSITY

UNIVERSITY SUPPLEMENTARY EXAMINATIONS

SECOND YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN (ELECTRICAL, CIVIL, MECHANICAL) ENGINEERING

ECU 200: ENGINEERING MATHEMATICS V

DATE : TIME: 2 HOURS

INSTRUCTIONS: Attempt Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a. State the fundamental theorem of Integral Calculus. (1 mark)

b. Find the following integrals:

i. $\int \frac{3x+1}{(x-1)(x^2+1)}dx$ (4 marks)

ii. $\int_0^{\pi} 6x\sin(x^2)dx$ (4 marks)

iii. $\int \sin^4 t \cos t dt$ (3 marks)

c. Find the number c that satisfies the mean value theorem of integral calculus for the function $y = \frac{1}{2x+1}$ over the interval [0 , 1]. (3marks)

d. Determine the double integral $\iint x^2 ydydx$. (2 marks)

e. Given $Ae^x + Be^{-x} = 4\cosh x - 5\sinh x$. Determine the values of A and B.

(3 marks)

f. Evaluate the area bounded by the curve $y = \sin x$, the x-axis and the lines $x = 0$ and $x = \frac{3\pi}{2}$. (4 marks)

g. Use Euler's method to obtain a numerical solution of the differential equation $\frac{dy}{dx} = 3(1+x) - y$ given the initial conditions that $x = 1$ when $y = 4$, for the range $x = 1.0$ to $x = 2.0$ with intervals of 0.2. (6 marks)

QUESTION TWO (20 MARKS)

a. Given $y = \frac{x}{cox}$, find $\frac{dy}{dx}$ and hence determine $\int (\sec x + x\tan x \sec x)dx$. (3 marks)

b. Evaluate the volume of revolution generated by rotating about the x-axis, the region bounded by the function $f(x) = e^x$ and the lines $x = 0$ and $x = \ln 2$.

(3 marks)

c. Express $\frac{x^3 + 2x^2 + x - 1}{x^2 + x - 2}$ in partial fractions and use your result to find the integral $\int \frac{x^3 + 2x^2 + x - 1}{x^2 + x - 2} dx$

(6 marks)

d. Using trapezoidal rule estimate $\int_1^4 \ln x dx$. Convert to four decimal places and take $r = 6$. (5 marks)

e. Prove the hyperbolic identity $\cosh^2 x - \sinh^2 x = 1$. (3 marks)

QUESTION THREE (20 MARKS)

a. Calculate the length of the arc of the curve $y = \sqrt{x}$ between $x = 0$ and $x = 3$.

(4 marks)

b. Use the substitution $t = \tan\frac{x}{2}$ to evaluate $\int_0^{\frac{\pi}{2}} \frac{1}{4+5\cos x} dx$ (5 marks)

c. Determine the value of the integral $\int_0^{\frac{4}{3}} \frac{1}{\sqrt{16-9x^2}} dx$ (4 marks)

d. Use tabular integration to evaluate $\int x^4 e^{-x} dx$. (3 marks)

e. A chain hangs so that its shape is of the form $y = 56\cosh\left(\frac{x}{56}\right)$. Determine, correct to 4 significant figures:

i. The value of y when x is 35. (2 marks)

ii. The value of x when y is 62.35. (2 marks)

QUESTION FOUR (20 MARKS)

a. A particle starts in motion with velocity of $10m/s^2$ and its acceleration after time t is given by $a = -2t + 3$. Find the distance travelled when the particle first comes to rest. (7 marks)

b. Evaluate the double integral $\int_0^{\pi}\int_2^4 x\sin y\,dxdy$ (3 marks)

c. Use integration by parts to find $\int x^2(\log_e x)^2 dx$ (4 marks)

d. Determine the power series for $2\cosh\left(\frac{\theta}{2}\right) - \sinh 2\theta$ as far as the term θ^5.

(3 marks)

e. Find the area enclosed by the curve $y = x(x-1)(x-2)$ and the axis between $x = 0$ and $x = 4$. (3 marks)

QUESTION FIVE (20 MARKS)

a. A yeast culture is growing at a rate of $\frac{d\mu}{dt} = 0.3e^{0.2t}$, where t is the time in hours, $\mu(t)$ is the mass in grams. At time $t = 0$ the mass was 0.1 grams.

i. Use Simpson's Rule with $r = 6$ to estimate the mass after 3 hours. (5 marks)

ii. Estimate the absolute value of the maximum error that can occur when growing the culture after 3 hours. (3 marks)

b. Evaluate $\int \frac{1}{\sqrt{9-4x^2}}\,dx$. (4 marks)

c. The region bounded by the curve $y = x^2$ and the line $y = x$ is rotated about the y-axis. Calculate the volume of the solid of revolution generated. (4 marks)

d. Find the average value f_{avg} of the function $y = 2 + \sin^2 x$ over the interval $\left[\frac{\pi}{2}, \pi\right]$.

Hence find a value such that $f(c) = f_{avg}$. (4 marks)