MACHAKOS UNIVERSITY

University Examinations for 2020/2021

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)

BACHELOR OF SCIENCE (CIVIL ENGINEERING)

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

ECU 200: ENGINEERING MATHEMATICS V

DATE: 12/8/2021 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS: Answer Question One and Any Other Two Questions

QUESTION ONE (30 MARKS)

a) Given $\int_{-1}^{1} g(x)dx = 5$ and $\int_{1}^{4} g(x)dx = -2$. Evaluate:

i. $\int_{4}^{1} g(x)dx$ (1 mark)

ii. $\int_{-1}^{4} g(x)dx$ (1 mark)

b) A chain hangs so that its shape is of the form $y = 40\cosh\left(\frac{x}{40}\right)$. Determine, correct to 4 significant figures:

i. The value of y when x is 25. (1 mark)

ii. The value of x when y is 54.30. (2 marks)

c) Determine the following integrals:

i. $\int_0^2 \frac{2x}{\sqrt[3]{x^2+1}}dx$ (4 marks)

ii. $\int \frac{1}{\sqrt{9-4x^2}}dx$ (4 marks)

d) Find the are below $f(x) = -x^2 + 4x + 1$ and above $g(x) = -x^3 + 7x^2 - 10x + 3$ over the interval $1 \leq x \leq 2$. (3 marks)

e) Compute the area of the surface formed when $f(x) = x^2$ between 0 and 2 is rotated around the y-axis (4 marks)

f) Evaluate the volume of revolution generate by rotating about the x-axis the region bounded by the function $f(x) = e^x$ and the lines $x = 0$ and $x = ln2$.

(3 marks)

g) Determine $\lim_{x \to 0} \left\{ \frac{x - sinx}{x - tanx} \right\}$ (4 marks)

h) Express the rational fraction $\frac{-x+1}{x^2+4x+3}$ as a sum of partial fractions. (3 marks)

QUESTION TWO (20 MARKS)

a) A beam 10 meters long has density $\sigma(x) = x^2$ at distance x from the left end of the beam. Find the center of mass $\bar{x}$. (3 marks)

b) The region bounded by the curve $y = x^2$ and the line $y = x$ is rotated about the y-axis. Calculate the volume of the solid of revolution generated. (4 marks)

c) Find the average value f_{avg} of the function $y = 2 + \sin^2 x$ over the interval $\left[\frac{\pi}{2}, \pi\right]$.

Hence find a value such that $f(c) = f_{avg}$. (4 marks)

d) Use trapezoidal rule to evaluate $\int_1^2 \frac{1}{x^2+1} dx$ Using 5 ordinates to evaluate to 5 d.p.

(4 marks)

e) Evaluate $\int \frac{x^2+3x-8}{x^2+2x-8} dx$ (5 marks)

QUESTION THREE (20 MARKS)

a) State Euler's (series) method. (2 marks)

b) Solve $\int_0^{\pi/4} \sqrt{1+\cos 4x}dx$. **Hint use** $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ (3 marks)

c) Evaluate $\int \sqrt{1+\sqrt{x}}\, dx$ (4 marks)

d) What is the coefficient of $x^{12}y^{13}$ in the expansion of $(x+y)^{25}$? (2 marks)

e) Evaluate $(1.002)^9$ using the binomial theorem correct to:

i. 3 decimal places. (4 marks)

ii. 7 significant figures. (1 mark)

f) Using Maclaurin's series, find the first five terms for the expansion of the function. $f(x) = e^{3x}$. (4 marks)

QUESTION FOUR (20 MARKS)

a) Using integration by parts, evaluate the following integral

i. $\int e^{3x} \sin(4x) dx$ (4 marks)

ii. $\int \ln(8x) dx$ (2 marks)

b) Suppose a sky diver leaps from a hovering helicopter and falls in a straight line. He falls at a terminal velocity of $80m/s$ for 19 seconds, at which time he opens his parachute. The velocity decreases linearly to $6m/s$ over a two-second period and then remains constant until he reaches the ground at $t = 40$ seconds. The motion is described by the velocity function:

$$V(t) = \begin{cases} 80 & if\ 0 \le t \le 19 \\ 783 - 37t & if\ 19 \le t \le 21 \\ 6 & if\ 21 \le t \le 40 \end{cases}$$

Determine the altitude from which the sky diver jumped. (4 marks)

c) Prove that $1 + 2\sinh^2 x = \cosh 2x$ (4 marks)

d) If $4e^x - 3e^{-x} \equiv P \sinh x + Q \cosh x$ determine the values of P and Q. (6 marks)

QUESTION FIVE (20 MARKS)

a) Compute the surface area of a sphere of radius r obtained by rotating the graph of $f(x) = \sqrt{r^2 - x^2}$ about the x-axis. (5 marks)

b) Use Euler's method to obtain a numerical solution of the differential equation $\frac{dy}{dx} + y = 2x$ given the initial conditions that $x = 0$ when $y = 1$, for the range $x = 0(0.2)1.0$. Draw the graph of the solution in this range. (8 marks)

c) Determine the first four terms of the power series for $Cos2\theta$. (3 marks)

d) Compute $\iint_R 1 + (x-1)^2 + 4y^2 dA$, where $R = [0,3]X[0,2]$, in two ways. (4 marks)