MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS

SMA 203 LINEAR ALGEBRA II

DATE: 22/3/2021 **TIME: 8.30-10.30 AM**

INSTRUCTIONS TO CANDIDATES

Attempt question ***one (compulsory)*** and any other ***two questions***.

QUESTION ONE (30MARKS)

a) State the Cayley Hamilton theorem (3 marks)

b) Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and let $g(x) = x^2 - 5x - 2$, $f(x) = 2x^2 - 3x + 7$. Show that A is a root of $g(x)$ and not a root of $f(x)$ (4 marks)

c) Express $(2, -5, 3)$ as a linear combination of $(2, -3, 2)$ $(2, -4, 1)$ and $(1, -5, 7)$ (4 marks)

d) Let $f: V \to V$ be a linear mapping. Let A be the matrix relative to a basis $v_1\ v_2\ v_3 \ldots .v_n$. Prove that the columns of A span $im\ f$ (4 marks)

e) Define a linear mapping (3 marks)

f) Let $v = \Re^4$; $w = \{(x, y, z, w) / x + y - z = 0\}$ by $f(x, y) = (x + y, 0)$.show that w is a subspace of $\Re^4$ (5 marks)

g) Prove that "any set of linearly independent vectors in V can be extended into a basis for V" (4 marks)

h) Let $V = P_3(x)$ and define $D: v \to v$ by $Df = \dfrac{df}{dx}$, D is linear. Determine

i) Nullity (2 marks)

ii) Rank (1 mark)

QUESTION TWO (20 MARKS)

a) Define $f:\Re^3 \to \Re^3$ by $f(x,y,z)=(x+y,z,-x)$. Let $S=\{(1,1,1),(1,2,1),(0,-1,1)\}$ be an ordered basis for $\Re^3$. Determine the matrix of f relative to the basis S. (8 marks)

b) If $f: v \to w \quad (v, w \text{ vector spaces})$ prove that;

i. $\ker f$ is a subspace of v (6 marks)

ii. $im\ f$ is a subspace of w (6 marks)

QUESTION THREE (20MARKS)

a) Determine the minimal polynomial of the matrix

$$A=\begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & 4 \end{pmatrix}$$

(7 marks)

b) Let V be a subspace of R^4 spanned by $(1,-1,0,0)$ $(0,0,1,-1)$ $(0,0,3,1)$ and let u be the subspace of R^4 spanned by $(1,2,-2,3)\,(3,0,2,-1)(2,1,0,1)$. Determine a basis for;

i) $U+V$ (6 marks)

ii) $U\ n\ V$ (7 marks)

QUESTION FOUR (20MARKS)

a) Consider the matrix $A=\begin{pmatrix} -1 & -5 \\ 10 & 14 \end{pmatrix}$. Work out the matrices P *and* P^{-1} such that $P\ A\ P^{-1}=D$. Where D is a diagonal matrix. (6 marks)

b) Prove the dimension's formula $Dim(u+v)=\dim u+\dim v-\dim(u\,n\,v)$ (7 marks)

c) Calculate the eigen values and the eigen vectors of the matrix

$$A=\begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}$$

(7 marks)

QUESTION FIVE (20MARKS)

a) Let $f:\Re^n \to \Re^m$ have the matrix $A=\begin{pmatrix} 2 & 1 & -1 & 3 \\ 3 & 0 & 1 & 4 \\ 1 & 2 & -3 & 2 \end{pmatrix}$ with respect to standard basis.

Determine;

i) The value of $m \ and \ n$ (3 marks)

ii) The $im\, f$ and $rank\ f$ (5 marks)

iii) The $\ker f$ and $nullity\ f$ (5 marks)

b) Let V be a vector space and let w be a fixed vector in V. Define

$W=\{set\ of\ all\, vectors\ in\ V\ which\ are \perp W\}$

$v_1 \perp v_2 \iff v_1\ is\ perpendicular\ to\ v_2$.

$w=\{v\in v\ /\ v\cdot w=0\}$. Show that w is a subspace of v. (7 marks)