MACHAKOS UNIVERSITY,

ECU 300: ENGINEERING MATHEMATICS IX

University Supplementary Examinations

Instructions: Answer Question (Compulsory) and any other 2 Questions.

Question one (30 marks)

a) Find unit vectors normal to the surfaces $x^2 + y^2 - z^2 + 3 = 0$ and $xy - yz + zx - 10 = 0$ at the point (3, 2, 4) and hence find the angle between the two surfaces at that point. 4marks

b) Prove that $\nabla^2\left(\frac{1}{r}\right) = 0$ where $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$ 5marks

c) Prove the vector triple product $\boldsymbol{b} \times (\boldsymbol{c} \times \boldsymbol{a}) = (\boldsymbol{b}.\boldsymbol{a})\boldsymbol{c} - (\boldsymbol{b}.\boldsymbol{c})\boldsymbol{a}$ 4marks

d) Determine the unit base vectors in the directions of the following vectors and determine whether the vectors are orthogonal:
$2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}, 4\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}, \boldsymbol{i} + 4\boldsymbol{j} + 2\boldsymbol{k}$ 4marks

e) Find the directional derivative of $\varphi = (x + 2y + z)^2 - (x - y - z)^2$ at the point (2,1,-1) in the direction of $\boldsymbol{A} = \boldsymbol{i} - 4\boldsymbol{j} + 2\boldsymbol{k}$. 4marks

f) Using Divergence theorem, evaluate $\iint_S \vec{A}.\vec{n}\, ds \text{ where } \vec{A} = 4x\,\vec{i} - 2y^2\,\vec{j} + z^2\,\vec{k}$ taken over the region bounded by $x^2+y^2=4$, z=0 to z=3. 4marks

g) Given the space curve $x = 2t, y = t^2 \text{ and } z = \frac{2}{3}t^3$ find;

(i) Curvature k

(ii) Torsion τ. 5marks

Question Two (20 marks)

a) Given that $\vec{R} = e^{-t}\boldsymbol{i} + \ln(t^2+1)\,\boldsymbol{j} + \tan t\,\boldsymbol{k}$, determine;

i) $\left|\frac{d\vec{R}}{dt}\right|$ at $t = o$ 3marks

ii) $\left|\frac{d^2\vec{R}}{dt^2}\right|$ at $t = 0$ 3marks

b) A particle moves along the curve $x = 2t^2, y = t^2 - 4t \; and \; z = 3t - 5$, determine the components of its velocity and acceleration at a time $t = 1$ in the direction of $\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}$ 7marks

c) Given that $\vec{A} = x^2yz\boldsymbol{i} - 2xz^3\boldsymbol{j} + xz^2\boldsymbol{k}$and $\vec{B} = 2z\boldsymbol{i} + y\boldsymbol{j} - x^2\boldsymbol{k}$, determine $\frac{\partial^2}{\partial x \partial y}(\vec{A} \times \vec{B})$ at $(1,0,-2)$ 7marks

Question Three (20 marks)

a) If V is a scalar field such that $V = u^2vw^3$ and scale factors are $h_u = 1, h_v = 2 \; and \; h_w = 4, determine \; \nabla^2 V$ at the point (2,3,-1). 5marks

b) Prove the following Frenet-Serret formulae:

$$\frac{d\vec{B}}{dS} = -T\vec{N}$$

7marks

c) If $\phi = 2xyz^2$ and **F**=xy**i**-z**j**+x^2**k** and C is the curve x=t^2, y=2t and z=t^3 from t=0 to t=1; evaluate the line integrals:

i) $\int \phi \, dr$

ii) $\int_c F \times dr$ 8marks

Question Four (20 marks)

a) Find the equation of the tangent plane to the plane surface $2xz^2 - 3xy - 4x = 7$ at the point $(2, -2, 3)$ 4marks

b) Find the directional derivative $\phi = x^2yz + 4xz^2$ at $(1, -2, 1)$ in the direction $2\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k}$ 4marks

c) Verify Green's theorem in the plane for $\oint_c (xy + y^2)dx + x^2\,dy$ where c is the closed curve of the region bounded by y=x and $y=x^2$. 6marks

d) Given that $\vec{v} = \vec{w} \times \vec{r}$, where $\vec{w} = w_1\boldsymbol{i} + w_2\boldsymbol{j} + w_3\boldsymbol{k}$ and $\vec{r} = \boldsymbol{x}\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$, prove that $\vec{w} = \frac{1}{2} curl\,\vec{v}$. 6marks

Question Five (20 marks)

a) Find the work done in moving a particle in a force field given by $\boldsymbol{F} = 3xy\boldsymbol{i} - 5z\boldsymbol{j} + 10x\boldsymbol{k}$ along the curve $x = t^2 + 1; y = 2t^2; z = t^3$ from t=1 to t=2. 6marks

b) If **F**=2xz**i**-x**j**+y^2**k** , evaluate $\iiint_V \vec{F} dv$ where V is the region bounded by the surfaces x=0, y=0, y=6, $z=x^2$ and z=4. 7marks

c) Evaluate $\iint_S \vec{A}.\vec{n}\, ds$ where A=zi+xj-3zy^2 and s is the surface of the cylinder $x^2+y^2=16$ included in the first octant between z=0 and z=5. 7marks