MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (PROGRAMMING AND STATISTICS)

BACHELOR OF SCIENCE (INFORMATION TECHNOLOGY)

BACHELOR OF EDUCATION (SPECIAL NEEDS)

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF SCIENCE (ACTUAL SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

BACHELOR OF ECONOMICS

SMA 433: PARTIAL DIFFERENTIAL EQUATIONS II

DATE: 9/12/2021 **TIME: 11.00-1.00 PM**

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Define a second order linear partial differential equation (3 marks)

b) State the fourier transform's convolution theorem (3 marks)

c) State the short form of a second order partial differential equation (3 marks)

d) State the three classes of the second order partial differential equation (3 marks)

e) Consider the equation $2u_{xx} + 3u_{xy} + u_{yy} = 0$.

i. Classify the equation (4 marks)

ii. Determine its characteristic equation (4 marks)

f) Determine the Laplace transform of the function $f(t) = \cos 2x$ (5 marks)

g) Determine the fourier transform of the function $f(x) = e^{-a|x|}$ (5 marks)

QUESTION TWO (20 MARKS)

a) Use the fourier transform method to solve the differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0; \qquad u(x, 0) = f(x), \quad y > 0$$ (8 marks)

b) Consider the differential equation

$$\frac{d^2 x}{dt^2} - 3\frac{dx}{dt} + 2x = 2e^{3t}; \quad t = 0, \quad x = 5, \quad \frac{dx}{dt} = 7$$

Solve by Laplace transform method (12 marks)

QUESTION THREE (20 MARKS)

Solve the wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$

Given that $u(x,0) = f(x)$ and $u_t(x,0) = g(x)$.

QUESTION FOUR (20 MARKS)

Determine the characteristic of the Tricomi equation $y u_{xx} + u_{yy} = 0$ in the lower-half plane $y < 0$.

Put this equation into canonical form in the upper half- plane $y > 0$.

QUESTION FIVE (20 MARKS)

a) An infinitely long string having one end at $x = 0$ is initially at rest on the $x - axis$. At $t = 0$;the end $x = 0$ begins to move along the $u - axis$ in a manner described by $u(0,t) = a\cos\sigma t$.

Determine the displacement $u(x,t)$ of the string at any point at any time. (10 marks)

b) Using Mong'es method obtain the integral of $q^2 r - 2pqs + p^2 t = 0$ in the form $y + xf(z) = F(z)$ (10 marks)