MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF EDUCATION

SMA 363 TESTS OF HYPOTHESIS I

DATE: 17/4/2023 **TIME: 11:00 – 1:00 PM**

INSTRUCTIONS

Answer Question ONE and ANY OTHER TWO

QUESTION ONE (30 MARKS)

a) Explain briefly what is meant by hypothesis testing as used in statistics. (2 marks)

b) Outline the general procedure used for hypothesis testing. (4 marks)

c) Differentiate between Type I and Type II errors as used in hypothesis testing. (4 marks)

d) Use the Neyman-Pearson Lemma to obtain the best critical region for testing $\theta = \theta_0$ against $\theta = \theta_0 < \theta_1$, in the case of a normal population $N(\theta, \sigma^2)$, where σ^2 is known. (7 marks)

e) A machine is considered to be unsatisfactory if it produces more than 8% defectives. It is suspected that the machine is unsatisfactory. A random sample of 120 items produced by the machine contains 14 defectives. Does the sample evidence support the claim that the machine is unsatisfactory? Use $\alpha = 0.01$. (5 marks)

f) In the academic year 1997–1998, two random samples of 25 male professors and 23 female professors from a large university produced a mean salary for male professors of \$58,550 with a standard deviation of \$4000 and an average for female professors of \$53,700 with a standard deviation of \$3200. At the 5% significance level, can you conclude that the mean salary of all male professors for 1997–1998 was higher than that of all female professors? Assume that the salaries of male and female professors are both normally distributed with equal standard deviations. (8 marks)

QUESTION TWO (20 MARKS)

a) A manufacturer of television screens has a production line that used to produce an average of 200 screens per day. Because of recently introduced government regulations, a new safety device is installed, which the manufacturer believes will reduce the average daily output. After installation of the safety device, a random sample of 15 days' production was recorded, as follows:

186	206	190	202	182
210	192	188	202	176
196	188	202	184	190

Assuming that the daily output is normally distributed, is there sufficient evidence to allow the manufacturer to conclude that the average daily output has decreased following installation of the safety device? $\alpha = 0.05$. (5 marks)

b) Let $X_1, \dots, X_n$ be a random sample from a Poisson distribution with mean λ. Derive the most powerful test for testing $H_0: \lambda = 3$ against $H_a: \lambda = 6$. (7 marks)

c) Given the frequency function:

$$f(x, \theta) = \frac{1}{\theta}, 0 \leq x \leq \theta$$

$$= 0, \text{ elsewhere}$$

and you are testing $H_0: \theta = 1$ versus $H_1: \theta = 2$ on the basis of a single observation x. Determine the sizes of type I and type II errors for the following critical regions

i. $0.5 \leq x$ (3 marks)

ii. $1 \leq x \leq 1.5$ (3 marks)

In each case, obtain the power function of the test. (2 marks)

QUESTION THREE (20 MARKS)

a) In attempting to control the strength of the wastes discharged into a nearby river, an industrial firm has taken a number of restorative measures. The firm believes that they have lowered the oxygen consuming power of their wastes from a previous mean of 450 manganate in parts per million. To test this belief, readings are taken on $n = 20$ successive days. A sample mean of 312.5 and the sample standard deviation 106.23 are obtained. Assume that these 20 values can be treated as a random sample from a normal population. Test the appropriate hypothesis. Use $\alpha = 0.05$. (6 marks)

b) Because of the impact of the global economy on a high-wage country such as the United States, it is claimed that the domestic content in manufacturing industries fell between 1977 and 1997. A survey of 36 randomly picked U.S. companies gave the proportion of domestic content total manufacturing in 1977 as 0.37 and in 1997 as 0.36. At the 1% level of significance, test the claim that the domestic content really fell during the period 1977–1997. (6 marks)

c) Let $X_1, \dots, X_n$ be a random sample from a $N(\mu, \sigma^2)$. Assume that σ^2 is known. Determine an appropriate likelihood ratio test to test $H_0: \mu = \mu_0$ vs $H_a: \mu \neq \mu_0$ at level α. (8 marks)

QUESTION FOUR (20 MARKS)

a) In a one-year investigation of claim frequencies for a particular category of motorists, there were 150 claims from the 500 policyholders aged under 25 and 650 claims from the 4,500 remaining policyholders. Assuming that the number of claims made by individual motorists in each category has a Poisson distribution, test at the 1% level whether the claim frequency is the same for drivers under age 25 and over age 25. (5 marks)

b) The intelligence quotients (IQs) of 17 students from one area of a city showed a sample mean of 106 with a sample standard deviation of 10 whereas the IQs of 14 students from another area chosen independently showed a sample mean of 109 with a sample standard deviation of 7. Is there a significant difference between the IQs of the two groups at $\alpha = 0.02$? Assume that the population variances are equal. (7 marks)

c) A random variable X is believed to follow an $Exp(\lambda)$ distribution. In order to test the null hypothesis $\mu = 20$ against the alternative $\mu = 30$, where $\mu = \frac{1}{\lambda}$, a single value is observed from the distribution. If this value is less than 28, H_0 is accepted, otherwise H_0 is rejected. Find the probabilities of

i. a Type I error

ii. a Type II error (8 marks)

QUESTION FIVE (20 MARKS)

a) A physician claims that the variance in cholesterol levels of adult men in a certain laboratory is at least 100. A random sample of 25 adult males from this laboratory produced a sample standard deviation of cholesterol levels as 12. Test the physicians claim at 5% level of significance. (4 marks)

b) A new diet and exercise program has been advertised as remarkable way to reduce blood glucose levels in diabetic patients. Ten randomly selected diabetic patients are put on the program, and the results after 1 month are given by the following table:

Before	268	225	252	192	307	228	246	298	231	185
After	106	186	223	110	203	101	211	176	194	203

Do the data provide sufficient evidence to support the claim that the new program reduces blood glucose level in diabetic patients? Take $\alpha = 0.05$. (8 marks)

c) The table shows the midterm and final examination test scores for 10 students from a differential equations class, where x denotes the midterm scores and y denotes the final scores.

x	68	87	75	91	82	77	86	82	75	79
y	74	89	80	93	88	79	97	95	89	92

At 95% confidence level, test whether X and Y are independent. (8 marks)