MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FIRST YEAR SPECIAL/ SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (ENVIRONMENTAL SCIENCE)

ENS 137: MATHEMATICS FOR ENVIRONMENTAL SCIENCE

DATE: 23/9/2019 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS: Answer Question ONE and any other TWO.

QUESTION ONE(COMPULSORY) (30 MARKS)

a) A function can be defined as a rule that assigns each input x to exactly one output $f(x)$.

i) The set of inputs and the set of outputs are respectively referred to as? (2 marks)

ii) Sketch a diagram to illustrate this concept of a function. (3 marks)

b) Given the functions $f(x) = 2x - 3$ and $g(x) = x^2 - 1$ determine the composite functions $f(g(x))$ and $g(f(x))$. (4 marks)

c) If $\lim_{x \to c} f(x)$ and $\lim_{x \to c} g(x)$ exists, then state the algebraic property of limits associated with each of the following: $f(x) \pm g(x)$, $f(x)g(x)$ and $\frac{f(x)}{g(x)}$ (3 marks)

d) It is estimated that t years from now the population of a certain estate in the periphery a major city will be p thousand people, where $p(t) = 20 - \frac{7}{t+2}$. A study conducted by the city planning department indicates that the average amount of waste will be w parts per thousand when the population is p thousand, where $w(p) = 0.4\sqrt{p^2 + p + 21}$.

i) Express w as a function of time t. (2 marks)

ii) What happens to the amount of waste w in the long run (as $t \to \infty$)? (3 marks)

e) Use the first principle of derivatives to determine the derivative of function $f(x) = \frac{1}{x^2}$. (4 marks)

f) Compute $f'(x)$ given that $f(x)=\dfrac{2x^2+5}{(1-x^2)^3}$ (5 marks)

g) Evaluate the anti-derivative $f(t)=e^{-3t}+t^3+\dfrac{1}{t}$. (4 marks)

QUESTION TWO (20 MARKS)

a) State the three conditions a function $f(x)$ must satisfy for it to be continuous at $x=c$. (3 marks)

b) Determine the following limits:

i) $\lim\limits_{x\to 1}\dfrac{x^2+4x-5}{x^2-1}$ (3 marks)

ii) $\lim\limits_{x\to \infty}\dfrac{x^2-2x+3}{2x^2+5x+1}$ (3 marks)

c) Show that the function $f(x)=\begin{cases}\dfrac{x^2-1}{x+1} & if\ \ x<-1\\ x^2-3 & if\ \ x\geq -1\end{cases}$ is continuous at $x=-1$ (5 marks)

d) An ecologist models the effect of introducing a predator fish into a lake to control the population of prey fish by the function $P(t)=\dfrac{t+1}{t^2+t+4}$ where P is the population of the prey fish (in thousands) t months after the predator is introduced.

i) At what rate is the population changing when the predator is introduced? (4 marks)

ii) At what time does the population begin to decrease? (2 marks)

QUESTION THREE (20 MARKS)

a) Obtain the derivative of the function $f(x)=\sqrt{\dfrac{3x+1}{2x-1}}$:

i) Using the first principle. (7 marks)

ii) Using a combination of the appropriate rules of differentiation. (4 marks)

b) The value (in thousands of shillings) of a matatu is modeled by $V(N)=\left(\dfrac{3N+430}{N+1}\right)^{\frac{2}{3}}$ where N is the number of hours the matatu is used each day. Given that the usage varies with time in such a way that $N(t)=\sqrt{t^2-10t+45}$ where t is the number of months the matatu has been operating.

i) What will be the value of the matatu 9 months from now? (2 marks)

ii) At what rate is the value of the matatu changing with respect to time 9 months from now? (7 marks)

QUESTION FOUR (20 MARKS)

a) Find the equation of the line tangent to the graph of implicit function $y^3 + x^2 - 3xy = 0$ at the point $\left(\frac{3}{2}, \frac{3}{2}\right)$ and passing through $y = 3$. (5 marks)

b) An epidemiologist determines that a particular epidemic spreads in such a way that t weeks after the outbreak, N hundred new cases will be reported, where $N(t) = \frac{5t}{12 + t^2}$.

i) Find $N'(t)$ and $N''(t)$. (4 marks)

ii) At what time is the reported number of new cases at its peak? What is the maximum number of reported new cases? (3 marks)

c) Find all the critical points of $f(x) = 2x^3 + 3x^2 - 12x - 7$ and use the second derivative test to classify each point as a relative maximum or minimum. (8 marks)

QUESTION FIVE (20 MARKS)

a) Evaluate the following anti-derivatives:

i) $\int y^3\left(2y + \frac{1}{y}\right)dy$ (2 marks)

ii) $\int e^t\left(e^t + 1\right)^2 dt$ (2 marks)

b) Kenya Wildlife Service determines that the population $P(t)$ of flamingos that remain in Lake Nakuru t months after a protection policy is enforced is changing at the rate $\frac{dP}{dt} = 200e^{0.1t} + 150e^{-0.03t}$. If the population was 200,000 flamingos at the time of instituting the policy, what will the population be 12 months later? (6 marks)

c) Sketch the curve of $f(x) = (x-1)^2(x+2)$. (10 marks)