MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (APPLIED PHYSICS AND TECHNOLOGY)

BACHELOR OF EDUCATION (SPECIAL NEEDS EDUCATION)

BACHELOR OF EDUCATION (SCIENCE)

SPH 301: QUANTUM MECHANICS I

DATE: 19/4/2023 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS:

- The paper consists of **two** sections.
- Section **A** is **compulsory** (30 marks).
- Answer any **two** questions from section **B** (each 20 marks).

SECTION A

QUESTION ONE (30 MARKS)

a) The interpretation of $|\Psi(\vec{r},t)|^2$.as a position probability density requires that the probability be conserved. Explain the meaning of this statement (2 marks)

b) Consider a square integrable wave function: $\Psi(\vec{r},t)$and a Hamiltonian operator H. What is the condition for the Hamiltonian to be Hermitian? (2 marks)

c) Show that the momentum operator $\hat{p}_x = -i\hbar\frac{\partial}{\partial x}$ is Hermitian. (5 marks)

d) State Pauli's exclusion principle to prove that an atomic shell with quantum number n can accommodate only $2n^2$ electrons (3 marks)

e) Define and explain the four electronic quantum numbers (4 marks)

f) Show that two wave functions ψ_1 and ψ_2 are orthogonal if they have different Eigen values a_1 and a_2 respectively where $a_1 \neq a_2$. (4 marks)

g) Show that the commutator of two Hermitian operators $\hat{A}$ and $\hat{B}$ is anti-Hermitian. (5 marks)

h) Discuss the two slit experiment that serves to demonstrate the wave particle duality of electromagnetic radiation (5 marks)

SECTION B

QUESTION TWO (20 MARKS)

a) What is a wave function? Discuss its role in the two slit experiment. (4 marks)

b) Discuss the meaning of Heisenberg uncertainty relationship, $\Delta x \Delta p_x \geq \hbar$. Explain its application in the two slit experiment. (5 marks)

c) Prove that the Eigen values of Hermitian operator $\hat{A}$ are real. (5 marks)

d) If ψ_1 and ψ_2 are eigenfunctions of an Hermitian operator, show that $\int \psi_1 \psi_2{}^* d\tau = 0$ (6 marks)

QUESTION THREE (20 MARKS)

a) Show that the energy of a particle in a wave packet is a function of momentum and is given by $E(p_x) = \frac{p_x{}^2}{2m}$ where the parameters have their usual meaning (4 marks)

b) Separate the time-dependent Schrödinger equation $-\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x,t)}{\partial x^2} + V\psi(x,t) = i\hbar \frac{\partial}{\partial t}\psi(x,t)$ into two equations, one equation depending on *x* alone and the other on *t* alone (6 marks)

c) If the wave function $\psi(x,t)$ is linear, show that the time-dependent Schrödinger equation is also linear in $\psi(x,t)$. (5 marks)

d) Show that the Schrödinger equation $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$ can be obtained from Helmholtz's equation $\frac{d^2\psi(x,t)}{dx^2} + k^2\psi(x) = 0$ by substituting $K = \frac{2\pi}{\lambda}$ and $\lambda = \frac{h}{p}$ (5 marks)

QUESTION FOUR (20 MARKS)

a) What is the essence of Quantum Mechanics? Explain using two examples which classical mechanics fails to account (4 marks)

e) b) If $\psi = Axe^{\frac{-x^2}{2}}$ is an eigenfunction of the operator $\widehat{H} = \frac{-d^2}{dx^2} + x^2$, using the eigenvalue equation $\widehat{H}\psi = \lambda\psi$ determine the value of the Eigen value λ. (5 marks)

c) Derive the infinite square-well energy quantization law $E = \frac{\pi^2 n^2 \hbar^2}{2ma^2}$ by using equation $p = \frac{h}{\lambda}$ and $a = n\frac{\lambda}{2}$ where $p = \frac{h}{\lambda}$ is de Broglie's relation and "a" is the width of the well.

(6 marks)

d) Given the wave function $\psi(x) = Axe^{-x^{2/2}}$ calculate the expectation value of x. (5 marks)

QUESTION FIVE (20 MARKS)

a) A particle bounces back and forth between the walls of one-dimensional box at $\pm\frac{a}{2}$. The wave function for the lowest energy state of the particle is $\psi(x,t) = Acos\left(\frac{x\pi}{a}\right)e^{\left(\frac{-iEt}{\hbar}\right)}$ in the region $-\frac{a}{2} < x < \frac{a}{2}$ and it is zero outside this region. Show that it is a solution to the Schrödinger equation in the region and determine the value of energy E for the lowest energy state (7 marks)

b) Determine the energy levels and the corresponding normalized Eigen functions of a particle in one-dimensional potential well of the form:
$V(x) = \infty$ for $x < 0$ or for $x > a$; and $V(x) = 0$ for $0 < x < a$ (8 marks)

c) Consider an operator $\hat{R} = \frac{-d^2}{dx^2}$ and the eigen value equation $\hat{R}u(x) = \lambda u(x)$. Write out the possible eigen functions and discuss the conditions under which they are well behaved (5 marks)