MACHAKOS UNIVERSITY

University Examinations for 2024/2025 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCES

THIRD YEAR FIRST SEMESTER EXAMINATION FOR DEGREE IN
BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)
BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)
BACHELOR OF SCIENCE (MATHEMATICS)

SMA 330: NUMERICAL ANALYSIS 1

DATE: 24TH JANUARY, 2025 **TIME: 2.00 PM – 4.00 PM**

INSTRUCTIONS:

- *Answer question **ONE (Compulsory)** and **any other TWO** questions*
- *All the symbols used here have their usual meaning in numerical analysis.*
-

QUESTION ONE *(COMPULSORY)* (30 MARKS)

a) Show that $y_{n-3} = \left(1 - \nabla\right)^3 y_n$, given that $\nabla y_n = y_n - y_{n-1}$ (4 marks)

b) Given that $\Delta = E - I$ prove that $\Delta = \frac{\delta^2}{2} + \delta\sqrt{1 + \frac{\delta^2}{4}}$ (5 marks)

c) Determine the root of $f(x) = 3x - \cos x - 1$ that lies in the interval $(0.5, 0.7)$, using Newton-Raphson method and correct to 4 decimal places. (6 marks)

d) Determine the Newton's backward difference interpolating polynomial for the following data; (6 marks)

x	0	1	2	3
y=f(x)	1	2	1	10

e) Show that the maxima and minima values of a function $y = f(x)$ are obtained by solving the expression; (4 marks)

$$\Delta y_0 + \frac{2p-1}{2}\Delta^2 y_0 + \frac{3p^2 - 6p + 2}{6}\Delta^3 y_0 = 0$$

f) Consider a curve $y = f(x)$ through the points with the following coordinates; (5 marks)

x	0	0.25	0.50	0.75	1.00
y=f(x)	1.0000	0.9896	0.9589	0.9089	0.8415

Evaluate the volume of the solid of revolution $\int_0^1 \pi y^2 dx$ using Simpson's 1/3 rule.

QUESTION TWO (20 MARKS)

a) The table below shows the cumulative frequency of the number of students y who scored marks less than x in a numerical analysis examination;

x	40	50	60	70	80
y=f(x)	31	73	124	159	190

i) Construct the forward difference table. (5 marks)
ii) Determine the number of students with marks less than $x = 45$. (5 marks)
iii) How many students got marks between 40 and 45? (1 marks)

b) Given the table below;

x	3	4	5	6	7	8
y	0.205	0.240	0.259	0.262	0.250	0.224

i) Determine the minimum value of y. (7 marks)
ii) Obtain the value of x for which y is minimum. (2 marks)

QUESTION THREE (20 MARKS)

a) Consider the integral $\int_0^1 \frac{dx}{1+x}$

i) Transform this integral into the form $\int_{-1}^1 \frac{du}{u+3}$ (3 marks)

ii) Use the 3-point Gauss-Legendre formula to evaluate $\int_{-1}^1 \frac{du}{u+3}$ (7 marks)

b) Given $\frac{dy}{dx} = x^2(1+y)$ and $y(1) = 1,\ y(1.1) = 1.233,\ y(1.2) = 1.548,\ y(1.3) = 1.979.$

Evaluate $y(1.4)$ by the Adams-Moulton method. (10 marks)

QUESTION FOUR (20 MARKS)

a) Evaluate $\int_0^1 \frac{dx}{1+x}$ correct to 3 decimal places using Romberg's method, taking $h = 0.5, 0.25, 0.125$ successively. (11 marks)

b) Use the modified Euler's method to find an approximate value of y when $x = 0.3$ given that $\frac{dy}{dx} = x + y,\ y(0) = 1$ taking $h = 0.1$ (9 marks)

QUESTION FIVE (20 MARKS)

a) Apply Runge-Kutta 4th order method to find an approximate value of y when $x = 0.2$ given that $\frac{dy}{dx} = x + y,\ y(0) = 1$. (8 marks)

b) A curve $y = f(x)$passes through the points $(0,18)$, $(1,10)$, $(3, -18)$ and $(6,90)$;

i) Determine the equation of the curve using Lagrange's formula. (10 marks)

ii) Obtain the slope of the curve at $x = 2$. (2 marks)