



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE),

BACHELOR OF SCIENCE (MATHEMATICS),

BACHELOR OF EDUCATION

SMA 332: METHODS OF APPLIED MATHEMATICS I

DATE: 23/3/2021

TIME: 2.00-4.00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

- State the general second order linear partial differential equation (P.D.E.) (2 marks)
- Hence state the three classes of that P.D.E. while giving an example equation of the canonical form in each class. (5 marks)
- Write the Fourier Sine series expansion of a function $f(x)$ over $[0, \pi]$, stating the expression of the Fourier coefficient. (3 marks)
- Transform the wave equation into a set of ordinary differential equations. (5 marks)
- Determine the Laplace transform of $f(x) = x^2$. (2 marks)
- Use the Laplace transforms to solve the equation $y' + 2y = 3x^2$, $y(0) = 1$. (5 marks)
- Given that $u = u(x, y)$, apply the change of variables $\theta = \theta(x, y)$, $\eta = \eta(x, y)$ to express the following in terms of the variables θ, η :
 - The second order partial derivative u_{xx} (4 marks)
 - The second order partial derivative u_{xy} (4 marks)

QUESTION TWO (20 MARKS)

- a) The Laplace transform of a function $u = u(x, t)$ is $L\{u(x, t)\} = \int_0^\infty e^{-st} u(x, t) dt$. Determine the Laplace transform of $\frac{\partial^2 u}{\partial t^2}$. (7 marks)
- b) Given the partial differential equation $u_{xx} + yu_{yy} + \frac{u_y}{2} = 0$, in the xy -plane:
- i) Determine the region in the xy -plane where it is hyperbolic (2 marks)
- ii) Obtain the canonical form associated with the hyperbolic case. (11 marks)

QUESTION THREE (20 MARKS)

- a) Sketch the function $f(x) = x, -2 < x < 2$ whereby $f(x+4) = f(x)$ and hence determine the appropriate Fourier series expansion of the function. (8 marks)
- b) Determine the inverse Laplace transform of $\frac{6}{(s^2 + 1)(s^2 + 9)}$ using the convolution theorem (6 marks)
- c) Solve by Laplace's method the initial value problem $y'' + 2y' + 2y = 0, y(0) = 1, y'(0) = -1$ (6 marks)

QUESTION FOUR (20 MARKS)

- a) Solve $u_t = \alpha u_{xx}, 0 < x < \pi$ subject to: $u(0, t) = u(\pi, t) = 0, t > 0$ by separation of variables $u(x, 0) = f(x), 0 < x < \pi$ method to obtain the particular solution when $f(x) = x$. (14 marks)
- b) Show that the set of functions $\{\cos n\pi, \sin m\pi\}$ is orthogonal over the interval $[-\pi, \pi]$ (6 marks)

QUESTION FIVE (20 MARKS)

- a) The displacement of a semi-infinite elastic beam is described $u_{tt} = a^2 u_{xx}, x > 0, t > 0$ by $u(0, t) = f(t), \lim_{x \rightarrow \infty} \frac{\partial u}{\partial x} = 0, t > 0$. Solve for $u(x, t)$ by the Laplace transform method. $u(x, 0) = 0, \frac{\partial u}{\partial t} \Big|_{t=0} = 0, t > 0$ (10 marks)
- b) Find the characteristics of the equation $4u_{xx} + 12u_{xy} + 9u_{yy} - 9u = 9$ and reduce it to the appropriate canonical form. (10 marks)