



MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year
SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

University Supplementary Examinations

SMA 434: GAS DYNAMICS

Instructions

Answer question **one** (**Compulsory**) and any other **two** questions.

Question one (30 marks)

a) Define the following terms;

i) Reversible process 2 marks

ii) Mach cone 2 marks

iii) Choked flow 2 marks

b) Compute the density of air at 8400 Nm^{-2} , 540 K .

Take the gas constant $R=53.3 \text{ Jkg}^{-1} \text{ K}^{-1}$ 2 marks

c) Considering a closed system, that executes a complete cycle, state the second law of thermodynamics 2 marks

d) Consider a gas flowing through a stream tube. Given that at one section of the tube, the gas has a temperature of 320 K and is moving with a velocity of 270 ms^{-1} , determine the temperature at another section of the tube where the pressure reduces and the speed

increases to 1200 ms^{-1} . Take the specific heat of the gas at constant pressure to be $c_p = 2295 \text{ J kg}^{-1} \text{ K}^{-1}$. 4 marks

- e) A gas has a static temperature of 320 K and stagnation temperature of 349 K. determine its Mach number and hence classify the disturbance as either subsonic or supersonic.

Take the ratio of specific heats to be $\gamma = 1.41$. 3 marks

- f) Consider a flow between a slot separated by two plates; the lower at $y = 0$ is stationary and the one at $y = h$, moving with a velocity U . Given that the velocity is only in the x direction and only a function of y , the flow has no imposed pressure gradient or body force, the flow has no acceleration, assuming constant viscosity and using the linear momentum principle, show that the flow is linear. 5 marks

- g) Consider air at a temperature of 720 K flowing through a converging –diverging nozzle which has a cross sectional area of 2 cm^2 . Given that the initial mass flow rate is 0.075 kgs^{-1} , the initial and exit pressures are 250 kPa and 180.5 kPa. Determine;

i) Mass flow rate at the exit, 3 marks

ii) Exit velocity. 5 marks

Take $\gamma = 1.4$, $R = 287 \text{ J kg}^{-1} \text{ K}^{-1}$ and $c_p = 1004.5 \text{ J kg}^{-1} \text{ K}^{-1}$.

Question Two (20 marks)

- a) Consider a flow between a slot separated by two plates; the lower at $y = 0$ is stationary and the one at $y = h$, moving with a velocity U . Given that the velocity is only in the x direction and only a function of y , the flow is driven by a pressure difference in the x direction such that at $x = 0, P = P_0$ and at $x = l, P = P_1$. If there is no body force, the

flow has no acceleration, assuming constant viscosity and using the linear momentum principle, determine the velocity profile parameterized by P_o, P, h, U and μ . 10 marks

- b) Consider a calorically perfect ideal gas. Using the equation of state and enthalpy show that for a gas undergoing an isentropic process between two planes 1 and 2, the internal energy change, e is given by; 7 marks

$$\Delta e = \frac{P_2 V_2 - P_1 V_1}{\gamma - 1}$$

- c) Show that the individual gas constant is a difference of the specific heats at constant pressure and volume for a given gas, $R = C_p - C_v$ 3 marks

Question Three (20 marks)

- a) Consider a calorically perfect ideal gas flowing through a duct of length 85m. At one end of the duct it has a cross section of area 0.05 m^2 , the gas has a pressure of 500 kPa, a temperature of 470 K and is flowing at a velocity of 180 ms^{-1} . If the gas is heated with a constant heat flux along the entire surface of the duct such that at the other end it has a pressure of 563 kPa, a temperature of 486K, determine;

- i) The mass flow rate, 4 marks
ii) Velocity of the gas at the other end. 4 marks

$$\text{Take } R = 287 \text{ J kg}^{-1} \text{ K}^{-1}$$

- b) Consider an airplane flying through still air at a velocity of 300 ms^{-1} . Assuming steady isentropic flow of a calorically perfect gas having and given that the ambient air is has a temperature of 300 K and a pressure of 150.4 kPa, determine;

- i) The Mach number and hence classify the disturbance, 4 marks

- ii) Temperature, pressure and density at the nose of the airplane. 8 marks

$$\text{Take } \gamma = \frac{7}{5} \text{ and } R = 287 \text{ J kg}^{-1} \text{ K}^{-1}$$

Question Four (20 marks)

- a) Considering a one dimensional flow of a calorically perfect ideal gas undergoing a steady flow, with no area change where viscous effects and wall friction have no time to influence the flow, heat conduction and wall heat transfer do not also have time to influence the flow, for a normal shock wave having a disturbance speed D

i) Derive the equation for the Rayleigh line. $P_2 - P_1 = \rho_1^2 D^2 \left[\frac{1}{\rho_1} - \frac{1}{\rho_2} \right]$ 4 marks

ii) Derive the Rankine-Hugoniot relation $h_2 - h_1 = \frac{1}{2}(P_2 - P_1) \left[\frac{1}{\rho_1} + \frac{1}{\rho_2} \right]$ 6 marks

- b) Considering air flow through a normal shock. Given that the upstream conditions are $u_1 = 700 \text{ s}^{-1}$, $T_{01} = 540$, $P_{01} = 800 \text{ kPa}$, using Rankine-Hugoniot relations determine the downstream conditions M_2 , u_2 , T_2 and P_{02} 10 marks

$$\text{Take } \gamma = 1.4, c_p = 1000 \text{ J kg}^{-1} \text{ K}^{-1} \text{ and } R = 287 \text{ J kg}^{-1} \text{ K}^{-1}$$

Question Five (20 marks)

- a) Consider a gas flowing over a wedge inclined at an angle of 20° to the horizontal. Considering a calorically perfect ideal gas. Given that the upstream conditions are $P_1 = 150 \text{ kPa}$, $T_1 = 320 \text{ K}$ and $M_1 = 3$. By considering Hodograph and characteristic relations, determine the downstream pressure and temperature for a rarefaction shock of; $\beta = 37.76^\circ$. 5 marks

b) Considering an oblique shock wave, if $\Delta\theta \rightarrow 0$, assuming a negligible entropy change relative to pressure and velocity, for an isentropic flow, derive the expression for the Prandtl-Mayer function, $v(M)$ 10 marks

c) Consider a calorically perfect ideal gas. Given that it has a temperature of 320 K and is moving with a velocity of 600 ms^{-1} , determine the Prandtl-Mayer function $v(M)$.

Take $\gamma = 1.4$ and $R = 287 \text{ Jkg}^{-1}\text{K}^{-1}$ 5 marks