MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FOURTH YEAR SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION

BACHELOR OF ARTS

SMA 203: LINEAR ALGEBRA 11

DATE: 25/9/2019 **TIME: 11:00 – 1:00 PM**

INSTRUCTIONS:

ANSWER QUESTIONS ONE AND ANY OTHER TWO.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Let $T: R^2 \to R^2$ be a transformation defined by $T(x,y)=(x,y,x+y)$ and $S: R^3 \to R^1$ be defined by $S(x,y,z)=2x+3y-4z$. Determine $S \circ T$. (4 marks)

b) Find the coordinate matrix of P relative to the standard basis for P_2 given that $P=1+x+4x^2$. (3marks)

c) State the Cayley -Hamilton theorem (2 marks)

d) Define a Minimal polynomial. (2 marks)

e) Find all the Eigen-values and the corresponding Eigen vectors for the matrix $A=\begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}$. (4 marks)

f) Show that $T: R^2 \to R^2$ defined by $T(x,y)=(x+y,x)$ is a linear transformation. (4 marks)

g) Find the basis and the dimension of the kernel of $T: R^3 \to R^3$ defined by $T(x,y,z)=(x+4y,y+z,x-4z)$ (5 marks)

h) Let $T : R^2 \to R^2$ defined by $T(3,1) = (2,-4)$ and $T(1,1) = (0,2)$. Find $T(x, y)$ and in particular $T(7,4)$ (4 marks)

i) State the two conditions that a transformation must satisfy for it to be said to linear. (2 marks)

QUESTION TWO (20 MARKS)

a. Consider the following two basis of $\Re^2$ for $T : \Re^2 \to \Re^2$ defined by $T(x, y) = (2x + 3y, 4x - 5y)$ as $S = \{(1,2),(3,5)\}$ and $S' = \{(1,-1),(1,-2)\}$

   i) Determine the matrix representation of T relative to the basis S and S' (4 marks)

   ii) Find the change of basis matrix P from S to S'. (3 marks)

   iii) Find the change of basis matrix Q from S' to S. (3 marks)

b. Let $T : R^3 \to R^2$ be a linear transformation such that $T(\mathbf{v}_1) = (1,0)$ $T(\mathbf{v}_2) = (2,-1)$ and $T(\mathbf{v}_3) = (4,3)$ where $\mathbf{v}_1 = (1,1,1)$, $\mathbf{v}_2 = (1,1,0)$ and $\mathbf{v}_3 = (1,0,0)$. Find $T(2,-3,5)$ (7 marks)

c. Find the ccordinate matrix of $X = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$ relative to the standard basis for $M_{3,1}$ (3 marks)

QUESTION THREE (20MARKS)

a. Find the coordinate matrix $[\mathbf{X}]_B$ of $\mathbf{X} = (1,2,-1)$ in $\Re^3$ relative to the non standard basis $\mathbf{B} = \{(1,0,1),(0,-1,2),(2,3,-5)\}$ (4 marks)

b. Given that $T : R^2 \to R^2$ defined by $T(x, y) = (3x + 3y, x + 5y)$. Find all the eigen values and a basis of each eigenspace . (5 marks)

c. Verify the Cayley –Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, hence compute A^{-1}.

(5 marks)

d. Reduce the matrix $\begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ to the diagonal form. (6 marks)

QUESTION FOUR (20MARKS)

a. Find the characteristic polynomial, eigen values and minimal polynomial of the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 4 \end{bmatrix}$$ (7marks)

b. Let $T:R^2 \to R^3$ defined by $T(x_1,x_2)=\{x_1+2x_2,-x_1,x_3\}$ and let also $B=\{\mathbf{u}_1,\mathbf{u}_2\}=\{(1,3),(-2,4)\}$ and $B'=\{\mathbf{u}_1',\mathbf{u}_2',\mathbf{u}_3'\}=\{(1,1,1),(2,2,0),(3,0,0)\}$ be the bases of the R^2 and R^3 respectively. Find the matrix of T relative to bases B and B' (7 marks)

c. Find the Kernel, range,rank and nullity fro the linear function $T:R^2 \to R^3$ given by $T(x,y)=(x,2y,x+y)$ (6 marks)

QUESTION FIVE (20 MARKS)

a. Let $T:R^2 \to R^2$ be defined by $T(x,y,z)=(2x-y-z,3x-2y+4z)$. Find the matrix representation of T relative to the basis $\{(1,1,1),(1,1,0),(1,0,0)\}$ for R^3 and $\{(1,3),(1,4)\}$ for R^2. (7 marks)

b. Consider the matrix $A=\begin{bmatrix}-1 & 2 & -2\\ 1 & 2 & 1\\ -1 & -1 & 0\end{bmatrix}$, reduce it to the diagonal form. (6 marks)

c. Find the transition matrix from B to B' and vice versa for the following bases for R^3
$B=\{(1,0,0),(0,1,0)(0,0,1)\}$ and $B'=\{(1,0,1),(0,-1,2),(2,3,-5)\}$ (7 marks)