MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

DECEMBER SESSION EXAMINATION FOR

BACHELOR OF EDUCATION

SMA 431: DIFFERENTIAL GEOMETRY

DATE: SCHOOL BASED **TIME:**

INSTRUCTION

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) If $u = (3t^2 + 1)i + (\sin t)j$ and $v = (\cos t)i + e^t k$. Find:

i. $\frac{d}{dt}(u \bullet v)$ (3 marks)

ii. $\frac{d}{dt}(u \times v)$ (3 marks)

b) If $\bar{a} = 2i + 3j + 4k$ and $\bar{b} = 5i - 6j + 7k$ evaluate $\bar{a}x\bar{b}$ (3 marks)

c) Find the parametric equations and a vector equation of a line whose Cartesian equations are given by

$\frac{x-1}{5} = \frac{y+1}{2} = \frac{z}{-5}$ (4 marks)

d) The equation of the two planes Π_1 and Π_2 are given by , $2x + 3y - 4z + 1 = 0$ and $3x - 5y + 6z + 8 = 0$ respectively. Calculate the angle between the two planes. (5 marks)

e) Find the equation of a plane passing through the point $(2,1,-3)$ given that it is perpendicular to the vector $5i + 6j + 7k$ (5 marks)

f) Find the length of the arc of the curve

$\bar{r} = \cos ti + e^t \sin tj + e^t k$ from $0 \le t \le \pi$ (7 marks)

QUESTION TWO (20 MARKS)

a) If $\overline{a} = 2i - 3j + 4k$, $\overline{b} = 5i - 6j + 7k$ and $\overline{c} = 2i + j + 2k$.

Find

i. $\overline{b} \times \overline{c}$ (3 marks)

ii. $\overline{a} \bullet (\overline{b} \times \overline{c})$ (3 marks)

iii. $\overline{a} \times (\overline{b} \times \overline{c})$ (3 marks)

iv. $\overline{b}(\overline{a} \bullet \overline{c}) - \overline{c}(\overline{a} \bullet \overline{b})$ (6 marks)

b) Determine the value of m such that the vectors $\overline{a} = 2i + j + 4k$, $\overline{b} = 3i + 2j + mk$ and $\overline{c} = i - 4j + 2k$ are coplanar (5 marks)

QUESTION THREE (20 MARKS)

Consider the surface $\overline{r} = ui + vj + (u^2 + v^2)k$

a) Find the vector in the direction of the tangent to the curve $u = u_o$ on the surface. (2 marks)

b) Find the vector in the direction of the tangent to the curve $v = v_o$ on the surface. (2 marks)

c) Determine a vector normal to the surface. (4 marks)

d) What is the unit vector normal to the surface. (2 marks)

e) Find the equation to the tangent plane at the point P on the surface where $u = -1, v = 1$.

(5 marks)

f) Equation to the normal line at P. (5 marks)

QUESTION FOUR (20 MARKS)

a) Determine the value of p so that $\overline{a} = 2i + 3j + k$ and $\overline{b} = pi - 2j - 2k$. (2 marks)

b) Find

i. A unit normal to the surface

$\phi(x, y, z) = xy - z^2 = 0$ at the point $(1,4,-2)$ (5 marks)

ii. The equation to the normal line (2 marks)

iii. The equation to the tangent plane at $(1,4,-2)$ (3 marks)

c) If $\overline{a} = a_1 i + a_2 j + a_3 k$, $\overline{b} = b_1 i + b_2 j + b_3 k$, and $\overline{c} = c_1 i + c_2 j + c_3 k$.

Show that $\overline{a} \times (\overline{b} \times \overline{c}) = \overline{b}(\overline{a} \cdot \overline{c}) - \overline{c}(\overline{a} \cdot \overline{b})$ (8 marks)

QUESTION FIVE (20 MARKS)

For a space curve

$\bar{r} = l\cos ti + l\sin tj + mtk$ where l and m are constants determine the following

a) The unit tangent $\bar{T}$ (3 marks)

b) The curvature K and the radius of curvature ρ (5 marks)

c) The unit normal $\bar{n}$ (3 marks)

d) The unit binormal $\bar{b}$ (4 marks)

e) The Torsion τ and the radius of the Torsion σ (5 marks)