



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (APPLIED PHYSICS AND TECHNOLOGY)

SMA 105: MATHEMATICS FOR SCIENCE

DATE: 25/2/2021

TIME: 8:30 – 10:30 AM

INSTRUCTIONS:

Answer Question One and Any Other Two Questions

QUESTION ONE (30 MARKS) (COMPULSORY)

- a) Simplify the following expression

$$\left(x^{\frac{1}{2}}\right)(xy)^{\frac{3}{2}}y^3 \quad (3 \text{ marks})$$

- b) Determine the number of permutations of the letters of the word

BRICARDIALEUKEMIASIS (3 marks)

- c) Show that $\binom{3}{2} + \binom{3}{1} = \binom{4}{2}$ (3 marks)

- d) Find the exact value of $\sin 105^\circ$. (3 marks)

- e) Determine $\log_2(8 \times 32)$ (3 marks)

- f) Find the modulus and argument of the complex number $z=2+i$ (3 marks)

- g) If $z_1 = [2, 45^\circ] = 2(\cos 45^\circ + i \sin 45^\circ)$ and $z_2 = [3, 15^\circ] = 3(\cos 15^\circ + i \sin 15^\circ)$. Find the product of $z_1 z_2$. (4 marks)

- h) Find the quotient $\{(6+2i)+(1+3i)\}/-1+i-2$ (4 marks)

- i) Find the values of the six trigonometric functions of θ if θ has its terminal ray in Quadrant III and $\sin \theta = -\frac{2}{3}$. (4 marks)

QUESTION TWO (20 MARKS)

- a) Without using a calculator, simplify the following expressions

$$\left(\frac{81}{16}\right)^{\frac{3}{4}} \quad (2 \text{ marks})$$

- b) Solve for x in the equation $2\sin^2 x + 3\cos x - 3 = 0$ (3 marks)

- c) Use the binomial Theorem to determine $(9.9)^5$ to the nearest whole number. (5 marks)

- d) If $x = \log_9 5$ and $y = \log_3 5$ find y in terms of x (5 marks)

- e) ABC is a triangle such that $AB = 12m$, $BC = 21m$ and $\angle B = 101^\circ$. determine the remaining sides and angles. (5 marks)

QUESTION THREE (20 MARKS)

- a) Divide the complex number

$$5 - 2i \text{ by } 4 - 2i \quad (2 \text{ marks})$$

- b) Without using tables or calculator valuate

$$\frac{\sin 45^\circ \cos 15^\circ + \sin 15^\circ \cos 45^\circ}{\sin 60^\circ \cos 15^\circ - \sin 15^\circ \cos 60^\circ} \quad (3 \text{ marks})$$

- c) Solve the equation, giving values of θ from 0° to 360° inclusive $3 - 3\cos\theta = 2\sin^2\theta$ (5 marks)

- d) If $\log_b M = 3$ and $\log_b P = 4$, determine $\log_b \frac{M^{\frac{1}{3}} P^{\frac{1}{4}}}{\sqrt{(M^2 P^3)}}$ (5 marks)

- e) Find the amplitude, phase angle displacement and vertical displacement for the graph of

$$y = 4 + \frac{3}{2} \cos \frac{1}{2}(\theta - 90^\circ). \quad (5 \text{ marks})$$

QUESTION FOUR (20 MARKS)

- a) In how many ways can four students be chosen from eight students and in how many of these ways will a particular student be included? (2 marks)

- b) Verify the given trigonometric identity

$$\frac{\sec^2 \theta - 1}{\sec^2 \theta} = \sin^2 \theta \quad (3 \text{ marks})$$

- c) Determine whether $x + 2$ is a factor of the polynomial $P(x) = x^4 - 7x^2 - 6x$. If so, find the other factors by long division (5 marks)

- d) Write down the equation, the sum and product of whose roots are respectively 3 and -2
(5 marks)
- e) Expand $(1-5x)^{\frac{1}{3}}$ in ascending powers of x up to and including the term in x^4 . State the range of values of x for which the expansion is valid. Use the expansion to obtain $\sqrt[3]{0.95}$ correct to 4 decimal places.
(5 marks)

QUESTION FIVE (20 MARKS)

- a) Solve the following quadratic equations using the method of completing the square
 $2x^2 - 6x - 3 = 0$
(3 marks)
- b) Write down the sum and product of the roots of the equation
 $3x^2 - 2x - 7 = 0$
(5 marks)
- c) Find the cube roots of unity and hence factorize $z^3 - 1$
(6 marks)
- d) Express $\frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x$ in the form $r\sin(x + \lambda)$ where r and λ are to be determined.
Hence solve the equation $\sin x - \sqrt{3}\cos x = 1$. for $0^\circ \leq x \leq 360^\circ$.
(6 marks)