MACHAKOS UNIVERSITY

University Examinations for 2024/2025 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCES

FIRST YEAR FIRST SEMESTER EXAMINATION FOR DEGREE IN
BACHELOR OF SCIENCE (STATISTICS & PROGRAMMING)
BACHELOR OF SCIENCE (MATHEMATICS & COMPUTER)
BACHELOR OF SCIENCE (MATHEMATICS)
BACHELOR OF EDUCATION (SCIENCE)

SMA 102: BASIC MATHEMATICS

DATE: 16TH JANUARY, 2025 **TIME: 8.30 AM – 10.30 AM**

INSTRUCTIONS:
Answer question ***ONE (Compulsory)*** *and* ***any other TWO*** *questions*

QUESTION ONE *(COMPULSORY)* (30 MARKS)

a) Define the following terms; (2 marks)
   i. Universal set
   ii. Cardinality of a set

b) Verify that the proposition $p \vee \sim (p \wedge q)$ (3 marks)

c) Prove that $(A \cup B)^c = A^C \cap B^c$ (5 marks)

d) Determine the roots of the equation $2z^2 + 2z + 5 = 0$, and illustrate your answer on an Argand diagram (5 marks)

e) When $x^3 - x^2 + ax + b$ is divided by $(x-1)$ the remainder is -8 and when the same expression is divided by (x+1) the remainder zero. Calculate the values of a and b, and factorize the expression completely. (5 marks)

f) Determine the coefficient of x^{10} in the expansion of $(2x-3)^{14}$ (3 marks)

g) Define each of the following as used in discrete counting;
   i. Permutation (1 mark)

ii. Combination (1 mark)

h) Determine the number of six-digit codes greater than 3000 which can be generated without repetition of digits, using the digits 1,2,3,4,5,6; if it must end with an even digit. (5 marks)

QUESTION TWO (20 MARKS)

a) Among 60 students in a class, 26 got A in CAT 1 and 21 got an A in CAT 2. If 17 students did not get an A in either CAT, how many students got A in both examination? (5 marks)

b) Given the conditional proposition $p \rightarrow q$, use truth table to determine which of $q \rightarrow p$, $\sim p \rightarrow \sim q$ and $\sim q \rightarrow \sim p$ is logically equivalent to $p \rightarrow q$ (5 marks)

c) Let $U = \{1,2,3,4,5,6,7,8,9\}$, $A = \{1,2,3,4\}$, $B = \{2,4,5,7\}$ and $C = \{3,4,5,6\}$. Find;

i. $(A \cap C)^c$ (2 marks)

ii. $(A \cap B)^c$ (2 marks)

iii. $(A \cup C)^c$ (2 marks)

iv. $(B - C)^c$ (2 marks)

v. $A \cup B \cup C$ (2 marks)

QUESTION THREE (20 MARKS)

a) Show by mathematical induction to show that the proposition $P(n) = 1+3+5+...+(2n-1) = n^2$ holds for all positive integers. (3 marks)

b) Solve the inequality $(x-6)(x+1) \succ 2x-12$ (5 marks)

c) Write the complex number $z = 4 - i4\sqrt{3}$ in polar form. (3 marks)
Hence find the values of each of the following leaving your answer polar form:

i. z^4 (2 marks)

ii. $\sqrt[3]{z}$ (2 marks)

d) Find the values of a and b if $(x-2)$ and $(x+1)$ are both factors of $ax^3 + 3x^2 - 9x + b$ and state the third factor of the expression. (5 marks)

QUESTION FOUR (20 MARKS)

a) A committee of 11 members is to be formed from 10 men and 6 women. Determine the number of ways in which it can be done to include at least 4 women. (5 marks)

b) Determine the first four terms in the binomial expansion of $\left(9 + \frac{x}{3}\right)^{\frac{1}{2}}$ and state its range of validity. (5marks)

c) Show that for small values of x;

i. $\sqrt{\frac{1-\frac{x}{4}}{1+\frac{x}{4}}} \approx 1-\frac{x}{4}+\frac{x^2}{32}$ (5 marks)

ii. By setting $x=\frac{1}{16}$, determine $\sqrt{\frac{63}{65}}$ approximately to four decimal places (5 marks)

QUESTIONS FIVE (20 MARKS)

a) Use mathematical induction to prove that the following proposition holds for all positive integers: $P(n)=1^2+3^2+5^2+...+(2n-1)^2=\frac{n^2(2n-1)(2n+1)}{3}$ (5 marks)

b) Determine the first four terms of the expansion of $(x+1)^{21}$ in ascending powers of x. Hence find the coefficient of x^3 in the expansion of $(2-3x)(x+1)^{21}$ (6 marks)

c) Use formal proof or Elements Argument Method to show that $(A\cap B)^c = A^C \cup B^c$ (5marks)

d) Use De Moivre's Theorem to determine the value of $\left(2\sqrt{3}+2i\right)^5$ leaving your answer in the form $x+iy$ (4marks)