



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF EDUCATION (SCIENCE & ARTS)

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 102: BASIC MATHS

DATE: 23/3/2021

TIME: 8.30-10.30 AM

INSTRUCTIONS TO CANDIDATES

Answer question ONE and any other TWO questions

QUESTION ONE (30 MARKS) (COMPULSORY)

- a) Let the universal set $U = \{x: x \in \mathbb{N}, x < 13\}$. Given that $E = \{1,6,7,9,11\}$, $F = \{4,6,8,9,10,12\}$ and $G = \{10, 6, 7\}$
- Display this information in a Venn diagram. (4 marks)
 - Using (i) above deduce the sets: $E \cup F \cup G$ and $E' \cap G$. (2 marks)
 - Write down the power set of the set G i.e. $P(G) = 2^G$. (3 marks)
- b) There are 25 students in a class and they study either Mathematics or Physics or both Mathematics and Physics. Let M be the set of students studying Mathematics and P the set of students who study Physics. Given that $n(M) = 12$ and $n(P) = 17$, find $n(M \cap P)$. (3 marks)
- c) A committee of 10 persons is to be chosen from nine men and six women. In how many ways can it be formed if at least four women are to be in the committee? (3marks)
- d) Simplify: $(2 - i)(5 + 3i)$, Leaving your answer in the form $z = r(\cos\theta + i\sin\theta)$. (5 marks)

- e) Use the factor theorem to test whether $(y + 3)$ is a factor of $2y^3 - 3y^2 - 32y - 15$ and find the other factors if any. (4 marks)
- f) Use Element argument method or Formal proof to show that $A \setminus B = A \cap B^c$ (3 marks)
- g) Use truth table to determine the validity of the argument: $[(P \rightarrow Q) \wedge \sim Q] \rightarrow \sim P$ (3 marks)

QUESTION TWO (20 MARKS)

- a) Use mathematical induction to prove that the following proposition:

$$P(n) = 1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}, \forall n \in \mathbb{N} \quad (4 \text{ marks})$$

- b) State the factor theorem then use it to show that $(x + 2)$ is a factor of $x^3 - 7x - 6$ and find the other two factors. (4 marks)
- c) Find the remainder when $f(x) = 6x^2 - 7x + 2$ is divided by $(2x - 1)$. (2 marks)
- d) State the remainder theorem then use it to determine the remainder when $f(x) = 6x^3 - 7x^2 - 16x + 12$ is divided by $(x - 3)$. (4 marks)
- e) Part of the expansion on $(h + kx)^5$ is $32 + 40x + 20x^2 + \dots$ Find the values of the constants h and k . (4 marks)
- f) Determine the coefficient of x^3 in the binomial expansion of $(2 - x)^8$. (2 marks)

QUESTION THREE (20 MARKS)

- a) Let $A = \{x, y, z\}$ and $B = \{1, x, z, 9\}$ and $C = \{g, h\}$. Find $(A \cap B) \times C$. (3 marks)
- b) Identify any three areas in real life where you can apply the knowledge of set theory. (3 marks)
- c) In an attempt to determine some off -the-job factors that might be indicators of on-the-job effectiveness, accompany made a study of 200 of its employees. They were interested in whether the employees had been recognized for superior work by their supervisors within the past year, whether they were involved in community activities, and whether they followed a regular exercise plan. The company found the following:
30 answered “Yes” to all the three

50 were recognized and they exercised

52 were recognized and were involved in community

77 were recognized

37 were involved in the community but did not exercise

95 were recognized or were involved in the community

95 answered “No” to all three

- i. Draw a well annotated Venn diagram that represents this information. (8 marks)
- ii. How many exercised only? (3 marks)
- iii. How many exercised or were involved in the community? (3 marks)

QUESTION FOUR (20 MARKS)

- a) Perform the following operations on complex numbers; in each case simplify as far as possible.
 - i. $(8 - 6i) + (-2 + 5i)$ (3 marks)
 - ii. $\frac{4-9i}{1+7i}$ (3 marks)
 - iii. $(3 - 4i)^2$ (3 marks)
- b) Convert the complex number $z = \sqrt{3} - i$ in the form $z = r(\cos\theta + i\sin\theta)$. Hence use De Moivre's theorem to find the value of z^3 . (5marks)
- c) Solve the quadratic equation: $x^2 + 2x + 5 = 0$, then illustrate your solutions on an Argand diagram. (6 marks)

QUESTION FIVE (20 MARKS)

- a) Differentiate between a permutation and a combination. (2 marks)
- b) Find the number of distinguishable ways of arranging the letters in the word.
MAMALANDLORD . (3 marks)
- c) Differentiate between a Tautology and a contradiction, hence determine whether the following is a Tautology, Contradiction or neither. $(P \Rightarrow Q) \wedge (\sim P \vee Q)$ (4 marks)
- d) Use mathematical induction to prove the proposition:

$$P(n) = \sum_{i=1}^n \frac{1}{i(i+1)} = \frac{n}{n+1}, \quad n \in \mathbb{N} \quad (4 \text{ marks})$$

- e) Convert the complex number $z = 2 - i\sqrt{3}$ in polar form, hence use De Moivre's theorem to find $(2 - i\sqrt{3})^4$. (4 marks)
- f) A mixed athletic team containing 4 women and 5 men is to be chosen from 8 women and 7 men. In how many ways can this be done? (3 marks)