MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FOUTH YEAR SUPPLEMENTARY EXAMINATION FOR BACHELOR OF ELECTRICAL AND ELECTRONICS ENGINEERING.

EEE402-COMPLEX ANALYSIS FOR ELECTRICAL ENGINEERS

DATE: 23/9/2019 **TIME: 11:00 – 1:00 PM**

INSTRUCTIONS:

Answer **Question One** and **Any Two** Other Questions.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Let $f(z)=\dfrac{z+1}{z^2+9}$

i) Determine the poles of $f(z)$ (2 marks)

ii) Calculate the residue s of $f(z)$ at its poles (2 marks)

b) Determine the Maclaurin's series expansion of $f(z)$ given by $f(z)=e^z$ (3 marks)

c) Evaluate $\lim\limits_{z\to 2}\dfrac{z^2-2z+4}{z-2}$ (3 marks)

d) Show that $z=0$ is a removable singularity of the function $f(z)=\dfrac{1-\cos z}{z^2}$. (5 marks)

e) Prove that the function $f(z)=z^2$ is analytic (5 marks)

f) Determine the Laurent series for $f(z)=\dfrac{1}{z(z+5)}$ valid in the region $|z|<5$ (5 marks)

g) Show that $f(z)=\dfrac{1}{2}$ is differentiable at all $z\neq 0$ and find its derivative. (5 marks)

QUESTION TWO (20 MARKS)

a) Show that $f(z)=e^{z}$ is a differentiable everywhere in the complex plane (4 marks)

b) Given that $w=f(x)=\left(z^{2}+1\right)^{\frac{1}{2}}$. Show that $z=\pm i$ are branch points of $f(x)$ (6 marks)

c) Evaluate the integral $\int_{C}\frac{5z-2}{z(z-1)}dz$, where C is the positively oriented circle $\lfloor z\rfloor=2$ (10 marks)

QUESTION THREE (20 MARKS)

a) Express the function $f(z)=z^{3}+z+1$ in the form $f(z)=u(x,y)+iv(x,y)$. (4 marks)

b) Find the residues of $f(z)=\frac{z+1}{z^{2}+9}$ at a singular points. (4 marks)

c) Determine the Laurent's series for $f(z)=e^{\frac{1}{z}}$ valid in the region $0<|z|<\infty$ (5 marks)

d) Evaluate the integral $\int_{C}\frac{z^{2}-z+1}{z-1}dz$, where C (a). $|z|=\frac{1}{2}$, and $|z|=2$, oriented in the positive sense. (7 marks)

QUESTION FOUR (20 MARKS)

a) Show that $u(x,y)=y^{3}-3x^{2}y$ is harmonic function throughout the plane and find a harmonic conjugate $v(x,y)$. (5 marks)

b) Solve for z, the equation $e^{z}=1+i$ (5 marks)

c) Use the power series expansions for e^{z} and $\frac{1}{1+z}$, to obtain the power series representation of $\frac{e^{z}}{1+z}$, in the region $|z|<1$. (5 marks)

d) Evaluate $\int_{C}\frac{e^{2z}}{(z-1)(z-2)}dz$, where C is a positively oriented circle $|z|=3$. (5 marks)

QUESTION FIVE (20 MARKS)

a) Determine the Taylor's series for $f(z)=\frac{1}{z+z^{4}}$ valid for $|z|<1$. (3 marks)

b) Expand $f(z)=\frac{3}{z^{2}(z-3)^{2}}$ in a Laurent series at $z=3$ (7 marks)

c) Find the Cauchy principal value of $\int_{-\infty}^{\infty}\frac{x^{2}+x}{(x^{2}+1)(x^{2}+4)}dx$ (10 marks)