MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS

SMA 431: DIFFERENTIAL GEOMETRY

DATE: 21/1/2021 **TIME: 2.00-4.00 PM**

INSTRUCTIONS: Answer **QUESTION ONE** and any other **TWO** of the remaining questions**.**

QUESTION ONE: COMPUSARY (30 MARKS)

a) Determine the values of a so that $\vec{A} = ai - 2j + k$ and $\vec{B} = 2ai + aj - 4k$ are orthogonal.

(3 marks)

b) Find the parametric equations and a vector equation of a line whose Cartesian equations are given by

$$\frac{x-1}{5} = \frac{y+1}{2} = \frac{z}{-5}$$ (4 marks)

c) If $\vec{A} = 3i - 2j + 4k$ and $\vec{B} = 2i - 3j - 2k$ determine the angle θ between the two vectors and the area of the triangle formed by vectors $\vec{A}$ and $\vec{B}$. (5 marks)

d) Find $\nabla \circ (\nabla \phi)$ where $\phi = \phi(x, y, z)$ is a differentiable scalar function (5 marks)

e) Given the space curve $x = 2t,\ y = t^2$ and $z = \frac{2}{3}t^3$. Find; Curvature k and Torsion τ

(6 marks)

f) Find the length of the arc of the curve $\bar{r} = \cos ti + e^t \sin tj + e^t k$ from $0 \le t \le \pi$ (7 marks)

QUESTION TWO (20 MARKS)

a) A straight line passes through the points $A(2,3,4)$ and $B(5,1,6)$

i. Write down the direction vector of $\overrightarrow{AB}$ (2 marks)

ii. State the vector equation of the line AB (2 marks)

iii. State the Cartesian equation of the line AB (2 marks)

iv. State the parametric equation of the line AB (2 marks)

b) Find the equation of a plane passing through the point $(2,1,-3)$ given that it is perpendicular to the vector $5i+6j+7k$ (3 marks)

c) If $\vec{A}=\left(y^2-x^2z^2\right)i+\left(x^2+y^2\right)j-x^2yzk$, determine $curl\vec{A}$ at the point $(1,3,-2)$ (3 marks)

d) Prove the vector triple product $\vec{A}\left(\vec{B}x\vec{C}\right)=\vec{B}\left(\vec{A}\circ\vec{C}\right)-\vec{C}\left(\vec{A}\circ\vec{B}\right)$ (6 marks)

QUESTION THREE (20 MARKS)

a) The vectors $\vec{a}=2i-j+k$, $\vec{b}=i+2j-3k$ and $\vec{c}=3i-pj+5k$ are coplanar. Find constant p (3 marks)

b) The equation of the two planes Π_1 and Π_2 are given by, $2x+3y-4z+1=0$ and $3x-5y+6z+8=0$ respectively. Calculate the angle between the two planes. (5 marks)

c) Find the length of the arc of the curve $\vec{r}=\cos ti+e^t\sin tj+e^tk$ from $0\leq t\leq\pi$ (5 marks)

d) Find the second fundamental form on the surface $\vec{r}=ui+vj+\left(u^2-y^2\right)k$ (7 marks)

QUESTION FOUR (20 MARKS)

a) Determine the value of p so that $\overline{a}=2i+3j+k$ and $\overline{b}=pi-2j-2k$. (2 marks)

b) Find the unit vector paralleled to the resultant of vectors $\vec{r_1}=2i+4j-5k$, and $\vec{r_2}=i+2j+3k$ (3 marks)

c) Determine the volume of the parallelepiped whose edges are given by $\vec{a}=2i-3j+4k$, $\vec{b}=i+2j-k$ and $\vec{c}=3i-j+2k$ (4 marks)

d) Consider the surface $\vec{r}=ui+vj+\left(u^2+v^2\right)k$

i. Find the equation to the tangent plane at the point P on the surface where $u=-1$ and $v=1$ (5 marks)

ii. Equation to the normal line at P (6 marks)

QUESTION FIVE (20 MARKS)

For a space curve

$\bar{r} = l\cos ti + l\sin tj + mtk$ where l and m are constants determine the following

a) The unit tangent $\overline{T}$ (3 marks)

b) The curvature K and the radius of curvature ρ (5 marks)

c) The unit normal $\overline{n}$ (3 marks)

d) The unit binormal $\overline{b}$ (4 marks)

e) The Torsion τ and the radius of the Torsion σ