



MACHAKOS UNIVERSITY

University Examinations for 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

SMA 436: METHODS OF APPLIED MATHEMATICS II

DATE: 8/12/2021

TIME: 8.30-10.30 AM

INSTRUCTIONS

Answer question one (**compulsory**) and any other **two** questions

QUESTION ONE (30 MARKS)

- a) Given that $\delta_j^i = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$ Expand $\delta_j^i x_i x_j$. (3 marks)
- b) If a_{ij} are constants and $\delta_{pq} = \frac{\partial x_p}{\partial x_q}$, calculate the partial derivative $\frac{\partial}{\partial x_k} (a_{ij} x_i x_j)$. (3 marks)
- c) Prove by tensor notation and the product rule for matrices that $(\mathbf{AB})^T = \mathbf{B}^T \mathbf{A}^T$ for any two conformal matrices $\mathbf{A}$ and $\mathbf{B}$. (4 marks)
- d) Define an Affine tensor and hence a Cartesian tensor. (2 marks)
- e) Find the Euclidean metric tensor in matrix form for spherical coordinates given that spherical coordinates x^i are connected to rectangular coordinates via $\bar{x}^1 = x^1 \sin x^2 \cos x^3$, $\bar{x}^2 = x^1 \sin x^2 \sin x^3$ and $\bar{x}^3 = x^1 \cos x^2$ (6 marks)
- f) Define the Christoffel symbols of first and second kind. (2 marks)
- g) Consider the initial value problem;

$$\frac{d^2 y}{dx^2} + xy = 1,$$

$$y(0) = 0, y'(0) = 0$$

Transform this initial value problem to a Volterra integral equation. (7 marks)

- h) Define a singular integral equation. (3 marks)

QUESTION TWO (20 MARKS)

- a) State the Quotient law. (2 marks)
- b) A quantity $A(p, q, r)$ is such that in the coordinate system x^i , $A(p, q, r)B_r^{qs} = C_p^s$, where B_r^{qs} is an arbitrary tensor and C_p^s is a known tensor. Prove that $A(p, q, r)$ is a tensor. (10 marks)
- c) Calculate the Christoffel symbols of the second kind for the Euclidean metric in polar given by;

$$G = \begin{bmatrix} 1 & 0 \\ 0 & (x^1)^2 \end{bmatrix} \quad (8 \text{ marks})$$

QUESTION THREE (20 MARKS)

- a) Suppose that in $\mathbb{R}^3$ a metric field is given in x^i by

$$g_{ij} = \begin{bmatrix} (x^1)^2 - 1 & 1 & 0 \\ 1 & (x^2)^2 & 0 \\ 0 & 0 & 64/9 \end{bmatrix} \text{ Where } [(x^1)^2 - 1](x^2)^2 \neq 1$$

- i) Show that if extended to all admissible coordinate systems according to the transformation law of covariant tensors, this matrix field is metric. (4 marks)
- ii) For the metric compute the arc-length parameter and hence determine the length of

$$\text{the curve; } C: \begin{cases} x^1 = 2t - 1 \\ x^2 = 2t^2 \\ x^3 = t^3 \end{cases}$$

For $0 \leq t \leq 1$ (8 marks)

- b) Determine the resolvent kernel $\Gamma(x, \xi; \lambda)$ associated with $K(x, \xi) = 1 - 3x\xi$ in the interval $(0,1)$ in the form of the power series in λ , obtaining the first three terms. (8 marks)

QUESTION FOUR (20 MARKS)

- a) Given the special coordinate system x^i defined from rectangular coordinates $\bar{x}^i$ by

$$x^1 = \bar{x}^1 \text{ and } x^2 = e^{\bar{x}^2 - \bar{x}^1};$$

- i) Compute the Euclidean metric tensor. (4 marks)
- ii) Given that $C: x^1 = 3t, x^2 = e^t, 0 \leq t \leq 2$, calculate the length of the curve. (5 marks)
- iii) Interpret b) above geometrically. (3 marks)
- b) Consider the boundary value problem

$$\frac{d^2y}{dx^2} + \lambda y = 0 ,$$

$$y(0) = 0 , y(l) = 0$$

Transform this boundary value problem to a Fredholm equation of the second kind.

(8 marks)

QUESTION FIVE (20 MARKS)

- a) Using Green's function transform the following problem into Fredholm integral equations.

$$y'' + xy = 1$$

$$y(0) = 0, y(1) = 1$$

(8 marks)

- b) Show that the Bessel equation

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (\lambda x^2 - 1)y = 0 ,$$

$$y(0) = 0 , y(1) = 0$$

transforms to the integral equations

$$y(x) = \lambda \int_0^1 G(x, \xi) \xi y(\xi) d\xi$$

Where

$$G(x, \xi) = \begin{cases} \frac{x}{2\xi} (1 - \xi^2), & x < \xi \\ \frac{\xi}{2x} (1 - x^2), & x > \xi \end{cases} \quad (12 \text{ marks})$$