MACHAKOS UNIVERSITY

University Examinations for 2020/2021

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

BACHELOR OF EDUCATION (SPECIAL NEEDS)

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

SMA 260: PROBABILITY AND STATISTICS I

DATE: 13/8/2021 **TIME: 2:00 – 4:00 PM**

INSTRUCTION: *Answer Question **ONE** which is compulsory and any other **TWO** Questions*

QUESTION ONE (30 MARKS)

a) Define the following terms

i) Random variable (2 marks)

ii) Probability mass function (2 marks)

b) A researcher wishes to determine the number of cups of coffee a customer drinks with an evening meal at a restaurant.

X	0	1	2	3	4
F(X)	0.31	0.73	0.94	0.98	1.00

i) Obtain the PDF of X (3 marks)

ii) Using the PDF obtained in (i), find the E(X) and Var(X) (7 marks)

c) Find the constant k such that the function below is a probability density function (4 marks)

$$f(x)=\begin{cases} kx^2, & 0<x<4 \\ 0, & \text{otherwise} \end{cases}$$

d) A continuous random variable X has moment generating function $M(t)=e^{8t^2+3t}$.
Determine the E (X) (3 marks)

e) A random variable X has density function given by;

$$f(x)=\begin{cases}2e^{-2x}, & x\geq 0\\ 0, & x<0\end{cases}$$

Find the $P(2\leq x\leq 3.5)$ to 4 decimal place. (4 marks)

f) Astronomers treat the number of stars in a given volume of space as a Poisson random variable. The density in the Milky Way Galaxy in the vicinity of our solar system is one star per 16 cubic light-years. What is the probability to 4 decimal place of four or more stars in 16 cubic light-years? (5 marks)

QUESTION TWO (20 MARKS)

a) State any two properties that a probability density function of continuous random variable X should satisfy. (2 marks)

b) In a certain country the probability of testing positive for corona virus is 0.4. If 12 individuals are selected at random, calculate the probability to four decimal place that
 (i) At most 3 tests positive (4 marks)
 (ii) More than 10 test positive (3 marks)

c) A random variable X is normally distributed with mean 50 and standard deviation 10.
Calculate $P(45\leq x\leq 62)$ (4 marks)

d) The length of stay at a hospital emergency department is the sum of the waiting and service times. Let X denote the proportion of time spent waiting and assume a beta distribution with $\alpha=3$ and $\beta=1$. Determine the P(X < 2.5) (7 marks)

QUESTION THREE (20 MARKS)

a) A continuous random variable X has a pdf f(x) given by

$$f(x)=\begin{cases}\dfrac{3}{64}x^2(4-x), & 0\leq x\leq 4\\ 0, & \text{elsewhere}\end{cases}$$

Determine the
 i) Mean of X (4 marks)
 ii) Cumulative distribution of X (7 marks)

b) Suppose that X has a hypergeometric distribution with $N = 20$, $n = 4$, and $K = 4$.
Determine the following to 4 decimal places
 i) $P(X\leq 2)$ (5 marks)
 ii) Variance of X (4 marks)

QUESTION FOUR (20 MARKS)

a) If X is a discrete random variable with a and b being constants, show that
$\operatorname{var}(ax+b) = a^2 \operatorname{var}(x)$ (5 marks)

b) Let the probability density function of X be given by

$$f(x) = \begin{cases} 6x(1-x), & 0<x<1 \\ 0, & \text{otherwise} \end{cases}$$

Find the probability density of $Y = X^3$ (7 marks)

c) The amount of time, in minutes, that a person must wait for a matatu in stage is uniformly distributed between zero and 16 minutes, inclusive.

Find the;

i) Mean, μ (2 marks)

ii) Standard deviation as a simplified fraction, σ. (3 marks)

iii) Ninety percent of the time, the time a person must wait falls below what value? This asks for the **90th percentile**. (3 marks)

QUESTION FIVE (20 MARKS)

a) State any two properties of a binomial distribution (2 marks)

b) Let X be a random variable with p.d.f given as

$$f(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

Find the moment generating function of X (6 marks)

c) The moment-generating function of a geometric random variable X is

$$M_x(t) = \frac{pe^t}{1-(1-p)e^t}$$

Use $M_x(t)$ to find the mean and variance of *X*. (12 marks)