MACHAKOS UNIVERSITY

University Examinations for 2020/2021

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

SMA 463 TIME SERIES ANALYSIS

DATE: 17/8/2021 **TIME: 8:30 -10:30 AM**

INSTRUCTIONS: Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Giving an example, explain one objective of time series analysis. (2 marks)

b) Explain briefly the four components of a time series. (8 marks)

c) Explain briefly the procedure you would adopt in the analysis of a time series. (4 marks)

d) Let $\{X_t\}, t = 0, \pm1, \pm2, \ldots$ be a stochastic process given by $X_t = a + bt + e_t$ where e_t is a sequence of independent random variables distributed with mean $\mu = 0$ and variance σ^2 as $e_t \sim N(0, \sigma^2)$. Show that $\nabla X_t = X_t - X_{t-1}$ is second order (weakly) stationary. (8 marks)

e) Explain briefly the importance of the periodogram in spectral analysis. (2 marks)

f) Consider the first-order moving average process defined by the equation

$X_t = Z_t + \theta Z_{t-1}, t = 0, \pm1, \ldots$ where $\{Z_t\} \sim WN\ (0, \sigma^2)$, θ is a real valued constant and $EX_t = 0$.

Determine;

i. $\gamma_x(t + h, t)$ (2 marks)

ii. The spectral density of the process. (4 marks)

QUESTION TWO (20 MARKS)

a) Explain briefly the importance of the correlogram in time series analysis. (2 marks)

b) Differentiate between strict stationarity and weak stationarity. (4 marks)

c) Suppose $Y_t = \beta_0 + \beta_1 t + X_t$, where $\{X_t\}$ is a zero-mean stationary series with auto-covariance function γ_k and β_0 and β_1 are constants. Show that $\{Y_t\}$ is not stationary but that $\nabla Y_t = Y_t - Y_{t-1}$ is stationary. (5 marks)

d) Consider the model given by $X_t = Z_t - 1.3Z_{t-1} + 0.4Z_{t-2}$where $Z_t$is a purely random process. Express the model using B notation and determine whether the model is stationary and/or invertible. (4 marks)

e) Consider the first order autoregressive process $X_t = \alpha X_{t-1} + e_t, -1 < \alpha < 1$. Show that the process can be expressed as an infinite MA process, hence find its autocorrelation function. (5 marks)

QUESTION THREE (20 MARKS)

a) Define an ARMA (p, q) process. (2 marks)

b) Explain briefly the three properties of the autocorrelation function. (3 marks)

c) Consider two MA(1) processes P and Q given by

$$P: X_t = Z_t + \theta Z_{t-1}$$

$$Q: X_t = Z_t + \frac{1}{\theta} Z_{t-1}$$

Obtain the autocorrelation function of each of the processes. Is a moving average process uniquely identified from a given autocorrelation function? Explain. (7 marks)

d) Given that Z_t is iid noise with zero mean and variance σ^2, find the autocorrelation function for the stationary process defined by $Y_t = 5 + Z_t - \frac{1}{2}Z_{t-1} + \frac{1}{4}Z_{t-2}$ (8 marks)

QUESTION FOUR (20 MARKS)

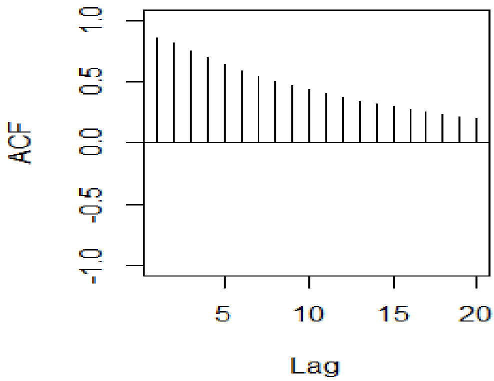
a) Describe the important characteristics of the autocorrelation function for the following models.

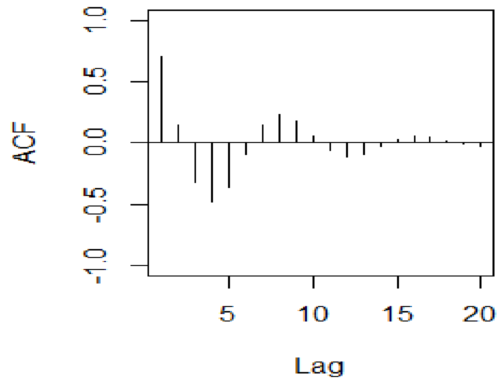
   i. MA(2)

   ii. AR(1)

   iii. ARMA(1,1) (3 marks)

b) Given the following ACF plots, explain with reason the generating process. (2 marks)





c) Consider the second order autoregressive process given by $Y_t = \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + e_t$ where e_t is a purely random process with mean zero and variance σ^2.

i. Derive the Yule-Walker equation for the process and its general solution. Find the values of α_1, α_2, such that the process is stationary. (8 marks)

ii. If $\alpha_1 = {}^1/_3$ and $\alpha_2 = {}^2/_9$, show that the autocorrelation function of X_t is given by $\rho(h) = \frac{16}{21}\left(\frac{2}{3}\right)^{|h|} + \frac{5}{21}\left(-\frac{1}{3}\right)^{|h|}, \;\; h = 0, \pm 1, \pm 2, \ldots$ (7 marks)

QUESTION FIVE (20 MARKS)

a) Define the spectral density function of a time series X_t and show that it can be expressed in terms of cosines. (4 marks)

b) Find the spectral density function of an AR (1) process given by $X_t = \alpha X_{t-1} + e_t$ (6 marks)

c) Determine if the following ARMA process is causal and if it is invertible. (5 marks)

$$X_t + 0.2\, X_{t-1} - 0.48 X_{t-2} = Z_t$$

d) Consider the random walk $S_t, t = 0, 1, 2, \ldots$ obtained by setting $S_0 = 0$ and $S_t = X_1 + X_2 + \cdots + X_t$, for $t = 1,2, \ldots$ where $\{X_t\}$ is iid noise with $E(X_t) = 0$ and $E(X_t^2) < \infty$. Determine;

i) $E(S_t)$ (1 mark)

ii) $E(S_t^2)$ (2 marks)

iii) $\gamma_s(t+h, t)$ (2 marks)