MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF ECONOMICS

BACHELOR OF EDUCATION

BACHELOR OF ARTS

SMA 305: COMPLEX ANALYSIS I

DATE: 23/4/2019 **TIME: 8:30 – 10:30 AM**

INSTRUCTION: *Answer Question **ONE** which is compulsory and any other **TWO** Questions*

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Find the fixed points of the bilinear transformation $w = \frac{3z-4}{z-1}$ (3 marks)

b) If $w = f(z) = 3z + 4 - 5i$, find the value of w which corresponds to

$z = -3 + i$ (3 marks)

c) Find the image of the point $(4,3)$ on $z-$plane under the transformation $w = 2z^2 + 3$ (4 marks)

d) Evaluate $\oint \frac{dz}{z-a}$ over a closed curve C and **a** is inside C (5 marks)

e) Simplify $\dfrac{(\cos 4\theta + i\sin 4\theta)(\cos 3\phi + i\sin 3\phi)}{(\cos\theta + i\sin\theta)(\cos 2\phi + i\sin 2\phi)}$ (5 marks)

f) Find the value of the derivative of $f = \frac{z-i}{z+i}$ at $z = i$ (5 marks)

g) Find the residue of $f(z) = \frac{z^2}{(z-1)(z+2)}$ at all its poles (5 marks)

QUESTION TWO (20 MARKS)

a) Consider the function $f(z) = \dfrac{z - \sin z}{z^2}$

i) State the singularity of $f(z)$ (2 marks)

ii) What kind of the singularity of $f(z)$ (2 marks)

iii) Find the Laurent series of $f(z)$ (5 marks)

iv) What is the region of convergence of $f(z)$ (1 mark)

b) Verify that $u = x^2 - y^2 - y$ is harmonic in the whole complex plane. Hence find harmonic conjugate function v of u. (10 marks

QUESTION THREE (20 MARKS)

a) Using definition of e^z prove that $e^{z_1}.e^{z_2} = e^{z_1+z_2}$ (5 marks)

b) Expand $f(z) = \frac{1}{(z-1)(z-3)}$ about $z = 1$ in Laurent series (5 marks)

c) Use residue theorem to show that $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta} = \frac{2\pi}{3}$ (10 marks)

QUESTION FOUR (20 MARKS)

a) Determine whether $f(z) = \frac{3z^4-2z^3+8z^2-2z+5}{z-i}$ is continuous at $z = i$. Hence determine its limit at $z = i$ (5 marks)

b) Prove that $\sin z = \sin x \cosh y + i \cos x \sinh y$ (5 marks)

c) Find the bilinear transformation which maps the points $z_1 = 2, z_2 = i, z_3 = -2$ into the points $w_1 = 1, w_2 = i, w_3 = -1$ respectively. (10 marks)

QUESTION FIVE (20 MARKS)

a) Determine the poles of $\frac{e^{iz}}{z^2(z^2+2z+2)}$ (5 marks)

b) Show that $f(z) = e^z$ is a periodic function with period $2k\pi i$ (5 marks)

c) Expand $f(z) = \frac{3}{z^2(z-3)^2}$ in a Laurent series at $z = 3$ (5 marks)

d) Evaluate $\int \bar{z}\, dz$ from $z = 0$ to $z = 4 + 2i$ along the line joining $z = 0$ to $z = 2i$ and the line joining $z = 2i$ to $z = 4 + 2i$ (5 marks)