MACHAKOS UNIVERSITY

ISO 9001:2015 Certified

UNIVERSITY EXAMINATIONS 2021/2022

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF PHYSICAL SCIENCES
FIRST **YEAR** ***FIRST SEMESTER*** **EXAMINATION FOR DEGREE IN BACHELOR OF EDUCATION (SCIENCE), BACHELOR OF SCIENCE (TELECOMMUNICATION AND INFORMATION TECHNOLOGY), BACHELOR OF SCIENCE (APPLIED PHYSICS AND TECHNOLOGY)**

SPH 103: MATHEMATICS FOR PHYSCIS

DATE: **TIME:**

INSTRUCTIONS TO CANDIDATES

Answer QUESTION ONE which is ***COMPULSORY*** and **ANY OTHER TWO** questions.

Question 1 carries **30** marks and the others carry **20** marks each.

You may find this useful		
$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$ $\int e^x\,dx = e^x + C$ $\int \sin x\,dx = -\cos x + C$ $\int \sec^2 x\,dx = \tan x + C$ $\int \sec x \tan x\,dx = \sec x + C$ $\int \sinh x\,dx = \cosh x + C$ $\int \tan x\,dx = \ln\lvert\sec x\rvert + C$ $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$ $\int a^x\,dx = \frac{a^x}{\ln a} + C$ $\int \cos x\,dx = \sin x + C$ $\int \csc^2 x\,dx = -\cot x + C$ $\int \csc x \cot x\,dx = -\csc x + C$ $\int \cosh x\,dx = \sinh x + C$ $\int \cot x\,dx = \ln\lvert\sin x\rvert + C$ $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C$	$\sin^2 x + \cos^2 x = 1$

QUESTION ONE (THIRTY MARKS)

a) Find the range and the domain of the function $y = \sqrt{4 - x^2}$ (**sketch a graph representing the domain and range**) (3 marks)

b) Find the roots for a quadratic equation $x^2 + x + 2 = 0$ in complex form (3 marks)

c) Solve for x in the equation $\log_2(x^3+5) = e^{\ln 5}$ (3 marks)

d) Find the first derivative $\left(\frac{dy}{dx}\right)$ of the function $y = \sin 3x^4$ (3 marks)

e) Using integration by parts, evaluate $\int x \sin x dx$ (4 marks)

f) Use partial fractions to decompose and evaluate $\int \frac{-x+4}{(x^2+1)(x-1)} dx$ (5 marks)

g) What is the probability of throwing a total of 6 points or less using three dices (3 marks)

h) Find the partial differentials $\frac{\partial R}{\partial x}$ for the function $R_{(x,y)} = \frac{x^2}{y^2+1} - \frac{y^2}{x^2+y}$ (3 marks)

i) Find the **angle** between vectors $\vec{A} = 2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$ and $\vec{B} = 7\underset{\sim}{i} + 24\underset{\sim}{k}$ (3 marks)

QUESTION TWO (TWENTY MARKS)

a) Evaluate the integral $\int e^x \cos x dx$ (7 marks)

b) Evaluate $\int_0^2 \frac{9a^2 da}{\sqrt[3]{a^3+8}}$ by substituting $u = a^3 + 8$. (**Write your answer to 3 significant figures**) (7 marks)

c) Evaluate the trigonometric integral $\int \sin^4 x \cos^7 x dx$ (6 marks)

QUESTION THREE (TWENTY MARKS)

a) A function F is defined for all real numbers t by the formula $F(t) = 2(t-1) + 4$. Evaluate F at the input value of 0, 2, $x+2$ and 3 (4 marks)

b) Use chain rule to evaluate $\frac{d}{dx}\left[\left(x^2+5\right)^3 + 3\right]^4$ (8 marks)

c) Obtain the second order derivative y'' of the function $y = xe^{2x^2} dx$ (8 marks)

QUESTION FOUR (TWENTY MARKS)

a) Graph the function $y = x^4 - 4x^3 + 10$ (8 marks)

b) Express $\frac{2x^3 - 4x^2 - x - 3}{x^2 - 2x - 3}$ as a sum of partial fractions and integrate (8 marks)

c) If 5 of 20 fuses in a box are, defective and 5 of them are randomly chosen for inspection, what is the probability that two of the chosen fuses are defective (4 marks)

QUESTION FIVE (TWENTY MARKS)

a) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the following function

$x^3 \sin\left(y^3\right) + xe^{3z} - \cos\left(z^2\right) = 3y - 6z + 2k$ (8 marks)

b) Express the product and quotient $\frac{z_1 z_2}{z_1}$ of the following complex numbers

$z_1 = -2 + 3i$ and $z_1 = 5 - i$ (6 marks)

c) Find $(\vec{A} \times \vec{B}) \times \vec{C}$, given that $\vec{A} = -\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$, $\vec{B} = \underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ and $\vec{C} = \underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$ (6 marks)