MACHAKOS UNIVERSITY

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE MATHEMATICS AND COMPUTER SCIENCE, BACHELOR OF MATHEMATICS

SMA 430. NUMERICAL ANALYSIS II

DATE: SEPT -DEC 2019 TIME:

INSTRUCTIONS: ANSWER QUESTION ONE (COMPULSORY) ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a. State any five direct methods of solving linear systems of equations (5 marks)

b. State any two iterative methods of solving linear systems of equations.(2 marks)

c. Solve the following system of equations using Gaussian elimination with partial pivoting

$$\begin{aligned} 8x_1 + x_2 + 8x_3 &= 49 \\ 9x_1 + x_2 + 2x_3 &= 33 \\ 5x_1 + 2x_2 + 8x_3 &= 52 \end{aligned}$$

(5 marks)

d. Determine the condition number using the maximum absolute column sum norm for the matrix A given that $A = \begin{bmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{bmatrix}$ (5 marks)

e. Obtain the Euclidean norm of the following vector and normalize it

$(2,3,4)$ (4 marks)

f. Obtain the spectral radius of matrix given by $A = \begin{bmatrix} 0 & 2 & 3 \\ -10 & -1 & 2 \\ -2 & 4 & 7 \end{bmatrix}$ (5 marks)

g. Solve the following system of equations using Gauss-Jordan method

$$x_1 + 3x_2 + 2x_3 = 17$$
$$x_1 + 2x_2 + 3x_3 = 16$$
$$2x_1 - x_2 + 4x_3 = 13$$

(4marks)

QUESTION TWO (20 MARKS)

a. Determine the Euclidean norm of the matrix A, where $A = \begin{bmatrix} 1 & 7 & -4 \\ 4 & -4 & 9 \\ 12 & -1 & 3 \end{bmatrix}$ (4 marks)

b. Solve the following system of equations using Cholesky's factorization method

$$2x - 6y + 8z = 24$$
$$5x + 4y - 3z = 2$$
$$3x + y + 2z = 16$$

(7 marks)

c. Using Euler's algorithm, solve the equation $\frac{dy}{dx} = 1 - y$, with the initial condition $x = 0$, $y = 0$ and tabulate the solutions at $x = 0.1, 0.2$ and 0.3 (9 marks)

QUESTION THREE (20 MARKS)

a. Given the following systems of equations

$$4x + y + 2z = 4$$
$$3x + 5y + z = 7,$$
$$x + y + 3z = 3$$

set up the Jacobi iterative scheme for the solution , iterate three times starting with the initial vector $x^{(0)} = 0$. Compare the result with the exact solution.

(10 marks)

b. Evaluate $y(0.4)$ by Milne's predictor –corrector method the differential equation $\frac{dy}{dx} = \frac{1}{2}(1 + x^2)y^2$ and $y(0) = 1$, $y(0.1) = 1.06$, $y(0.2) = 1.12$, $y(0.3) = 1.21$ (10 marks)

QUESTION FOUR (20 MARKS)

a. Solve the following system of equations using Crout's method

$$x + y + z = 9$$
$$2x - 3y + 4z = 13$$
$$3x + 4y + 5z = 40$$

(5 marks)

b. Use the method of least squares to fit the curve $y = \frac{a}{x} + \frac{b}{\sqrt{x}}$ to fit the table of values:

x	0.1	0.2	0.4	0.5	1	2
y	21	11	7	6	5	6

(5 marks)

c. Using Runge-Kutta method, find an approximate value of y for $x = 0.2$, if $\frac{dy}{dx} = x + y^2$ given that $y = 1$ when $x = 0$ (5 marks)

d. Obtain a linear polynomial approximation to the function $f(x) = x^3$ on the interval $[0,1]$ using the least square approximation with $w(x) = 1$ (5 marks)

QUESTION FIVE (20 MARKS)

a. Determine the solution of the following system of equations using Gauss-Seidal iteration method

$$8x + 2y - 2z = 8$$
$$x - 8y + 3z = -4$$
$$2x + y + 9z = 12$$

(10 marks)

b. Given $\frac{dy}{dx} = 1 + xy$ with initial condition that $y = 1$, $x = 0$. Compute $y(0.1)$ correct to four decimal places by using Taylor's series method (10 marks)