



MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

.....YEAR SEMESTER SPECIAL /SUPPLEMENTARY EXAMINATION FOR
BACHELOR OF

SMA 331: DYNAMICS II

DATE:

TIME:

INSTRUCTIONS TO CANDIDATES

Answer question **ONE (compulsory)** and any other **TWO** questions

QUESTION ONE (30 MARKS)

- a) State some important properties of a central force fields (3 marks)
- b) Prove that if a particle moves in a central force field, then its path must be a plane curve (5 marks)
- c) Given $m(\mathbf{r}\ddot{\theta} + 2\dot{\mathbf{r}}\dot{\theta}) = 0$, show that the equation of motion for a particle in a central field is $\mathbf{r}^2\dot{\theta} = h$, a constant (4 marks)
- d) Show by means of substitution $\mathbf{r} = \frac{1}{u}$ that the differential equation for the path of the particle in a central field is $\frac{d^2u}{d\theta^2} + u = -\frac{f(\frac{1}{u})}{mh^2u^2}$ (8 marks)
- e) Calculate the angular speed of the earth about its axis (4 marks)
- f) Two particles of masses m_1 and m_2 respectively are connected by a rigid massless rod of length, a and move freely in a plane. Show that the moment of inertia of the system about an axis perpendicular to the plane and passing through the center of mass is μa^2 where the reduced mass $\mu = \frac{m_1 m_2}{(m_1 + m_2)}$ (6 marks)

QUESTION TWO (20 MARKS)

- a) An observer stationed at a point which is fixed relative to an xyz coordinate system with origin O observes a vector $\mathbf{A} = A_1\mathbf{i} + A_2\mathbf{j} + A_3\mathbf{k}$ and calculates its time derivative to be $\frac{dA_1}{dt}\mathbf{i} + \frac{dA_2}{dt}\mathbf{j} + \frac{dA_3}{dt}\mathbf{k}$ later he finds that he and his coordinate system are actually rotating with respect to an XYZ coordinate system taken as fixed in space and having origin also at O. He asks “what would be the time derivative of $\mathbf{A}$ for an observer who is fixed relative to the XYZ coordinate system?” if $\frac{d\mathbf{A}}{dt}\bigg|_F$ and $\frac{d\mathbf{A}}{dt}\bigg|_M$

$\frac{d\mathbf{A}}{dt}\bigg|_F$ $\frac{d\mathbf{A}}{dt}\bigg|_M$

denote respectively the time derivatives of $\mathbf{A}$ relative to the fixed and moving systems, show that there exists a vector quantity $\boldsymbol{\omega}$ such that

$$\frac{d\mathbf{A}}{dt}\bigg|_F = \frac{d\mathbf{A}}{dt}\bigg|_M + \boldsymbol{\omega} \times \mathbf{A} \quad (10 \text{ marks})$$

- b) Find the moment of inertia and the radius of gyration of a solid circular cylinder of radius, a , height, h and mass, m about the axis of the cylinder using double integration (10 marks)

QUESTION THREE (20 MARKS)

An xyz coordinate system is rotating with respect to an XYZ coordinate system having the same origin and assumed to be fixed in space i.e. it is an inertial system. The angular velocity of the xyz system relative to the XYZ system is given by $\boldsymbol{\omega} = 2t\mathbf{i} - t^2\mathbf{j} + (2t + 4)\mathbf{k}$ where t is the time. The position vector of a particle at time t as observed in the xyz system is given by $\mathbf{r} = (t^2 + 1)\mathbf{i} - 6t\mathbf{j} + 4t^3\mathbf{k}$ at time $t = 1$, find the magnitude of the following quantities:-

- a) The apparent velocity given by $\frac{d\mathbf{r}}{dt}$ (2 marks)
- b) The true velocity given by $\frac{d\mathbf{r}}{dt} + \boldsymbol{\omega} \times \mathbf{r}$ (5 marks)
- c) the apparent acceleration given by $\frac{d^2\mathbf{r}}{dt^2}$ (2 marks)
- d) The true acceleration of the particle given by $\frac{d^2\mathbf{r}}{dt^2} + 2\boldsymbol{\omega} \times \frac{d\mathbf{r}}{dt} + \frac{d\boldsymbol{\omega}}{dt} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$ (6 marks)
- e) The Coriolis acceleration given by, $2\boldsymbol{\omega} \times \frac{d\mathbf{r}}{dt}$ (5 marks)

QUESTION FOUR (20 MARKS)

- a) Prove that $\frac{\partial \dot{\mathbf{r}}_v}{\partial \dot{q}_\alpha} = \frac{\partial \mathbf{r}_v}{\partial q_\alpha}$ where $\mathbf{r}_v = \mathbf{r}_v(q_1, q_2, \dots, q_n, t)$ (3 marks)
- b) Let $\mathbf{F}_v$ be the net external force acting on the v^{th} particle of a system. Prove that
$$\frac{d}{dt} \left\{ \sum_v m_v \dot{\mathbf{r}}_v \cdot \frac{\partial \mathbf{r}_v}{\partial \dot{q}_\alpha} \right\} - \sum_v m_v \dot{\mathbf{r}}_v \cdot \frac{\partial \dot{\mathbf{r}}_v}{\partial \dot{q}_\alpha} = \sum_v \mathbf{F}_v \cdot \frac{\partial \mathbf{r}_v}{\partial q_\alpha}$$
 (6 marks)
- c) Prove the Lagrange's equations,
$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_\alpha} \right) - \frac{\partial T}{\partial q_\alpha} = \phi_\alpha, \alpha = 1, 2, \dots, n$$
 Where
$$\phi_\alpha = \sum_v \mathbf{F}_v \cdot \frac{\partial \mathbf{r}_v}{\partial q_\alpha}$$
 (5 marks)
- d) If the Hamiltonian $H = \sum p_\alpha \dot{q}_\alpha - L$, where the summation extends from $\alpha = 1$ to n , is expressed as a function of the coordinates, q_α and moments, p_α prove the Hamilton's equations
$$\dot{p}_\alpha = -\frac{\partial H}{\partial q_\alpha} \text{ and } \dot{q}_\alpha = \frac{\partial H}{\partial p_\alpha} \text{ regardless whether } H$$
- i. Does not contain the variable time, t explicitly (3 marks)
- ii. Does contain the variable time, t explicitly (3 marks)

QUESTION FIVE (20 MARKS)

- a) Using the calculus of variations and the Hamilton's principle, prove that a necessary condition for $I = \int_a^b \mathbf{F}(x, y, y') dx$ to be an extremum (maximum or minimum) is $\frac{d}{dx} \left(\frac{\partial \mathbf{F}}{\partial y'} \right) - \frac{\partial \mathbf{F}}{\partial y} = 0$ (5 marks)
- b) Obtain the equation of motion $\ddot{\theta} + \frac{Mga}{I_0} \sin \theta$ for a compound pendulum where the force $Mg = -Mg\mathbf{j}$ is conservative so that the potential energy V is such that $-\nabla V = -\frac{\partial V}{\partial x} \mathbf{i} - \frac{\partial V}{\partial y} \mathbf{j} - \frac{\partial V}{\partial z} \mathbf{k}$ (5 marks)

- c) Find the moment of inertia of a right circular cone of height, h and radius, a about its axis and the radius of gyration, K using triple integration given

$$I = \int_{\theta=0}^{2\pi} \int_{r=0}^a \int_{z=0}^{\frac{h(a-r)}{a}} \sigma r^3 dr d\theta dz$$

and

$$M = \int_{\theta=0}^{2\pi} \int_{r=0}^a \int_{z=0}^{\frac{h(a-r)}{a}} \sigma r dr d\theta dz$$

(10 marks)

*******END*******