



MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF EDUCATION

SMA330: NUMERICAL ANALYSIS I

DATE: 20/1/2021

TIME: 8.30-10.30 AM

INSTRUCTIONS

Attempt question *one (compulsory)* and any other *two questions*.

QUESTION ONE (30 MARKS)

a) Given that $f(-2) = 12$, $f(-1) = 16$, $f(0) = 15$, $f(1) = 18$, $f(2) = 20$;

i.) Form the central difference table (5 marks)

ii.) Determine the value of $\delta_{y, \frac{3}{2}}^3$ (5 marks)

b) Evaluate $f(10)$ given that $f(x) = 168, 192, 336$ at $x = 1, 7, 15$ respectively. Use Lagrange's interpolation. (5 marks)

c) Consider the table below

x	0	1	2	3	4
$f(x)$	1	0	1	10	33

Determine the cubic polynomial interpolating the functional values above, (5 marks)

d) Obtain the approximate value of $I = \int_{-1}^1 e^{-x^2} dx$ using 2 point Gauss- Legendre integration method (5 marks)

e) Consider the equation $x^4 - x - 10 = 0$. Determine its root correct to three decimal places with $x_0 = 1.5$, $f(x_0) = -6.4375$, $f'(x_0) = 12.5$ using Newton Raphson method. (5 marks)

QUESTION TWO (20MARKS)

The population of a town in the decimal census was given as below.

<i>year</i>	1891	1901	1911	1921	1931
$f(x)$	46	66	81	93	101

Estimate the population for;

- a) 1895 (10 marks)
- b) 1925 (10 marks)

QUESTION THREE (20 MARKS)

- a) Consider the data below;

x			0	1	2	3
$f(x)$			1	0	1	10

- i. determine the Newton's forward difference interpolating polynomial for the data (5 marks)
- ii. work out $f(3.5)$ (3 marks)
- b) The elevation above a datum line of seven points of a road under construction are given as in the table below.

x	0	300	600	900	1200	1500	1800
y	135	149	157	183	201	205	193

Calculate;

- i. The gradient of the road at the middle point (8 marks)
- ii. The acceleration of any particle at the middle of the road (4 marks)

QUESTION FOUR (20 MARKS)

- a) Use **Radau three-point** integration to determine the value of $\int_0^1 \frac{1}{2x^2 + 2x + 1} dx$ (10 marks)
- b) Consider the differential equation $y'' + xy' + y = 0$, $y(0) = 1$, $y'(0) = 0$. Use Runge – Kutta method to calculate $y(0.1)$. (10 marks)

QUESTION FIVE (20 MARKS)

- a) Using the method of undetermined coefficients work out the nodes and weights of the quadrature formula for;

$$\int e^{-x} f(x) dx = \lambda_0 f(x_0) + \lambda_1 f(x_1) \quad (10 \text{ marks})$$

- b) Evaluate $\int_0^1 x\sqrt{1+x^2} dx$ using Romberg integration. (10 marks)