MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR SCIENCE (STATISTICS AND PROGRAMMING

BACHELOR SCIENCE (MATHEMATICS)

BACHELOR EDUCATION (SCIENCE)

SMA 361: THEORY OF ESTIMATION

DATE: 3/8/2017 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS

Answer question ONE (Compulsory) and any other TWO questions

QUESTION ONE (COMPULSORY)

a) A random sample of five values is taken from a normal population with mean μ and variance δ^2both unknown. The values are: 1.07, 1.02, 1.04, 1.06, and 1.05. Estimate the population mean and population variance. (3 marks)

b) $Let\ x_1, x_2, .. x_n$be a random sample drawn from a population with mean μ (unknown).Show that if $y = \sum_1^n a_i x_i$ is unbiased estimator for μ if $\sum_1^n a_i = 1$ (4 marks)

c) $Let\ x_1, x_2, .. x_n$be a random sample from a population with mean μ and variance δ^2. Consider the following estimators for μ.

$$t_1 = {}^1\!/_2\,(x_1 + x_2)$$

$$t_2 = \frac{\frac{1}{2}x_1 + x_2 + \cdots + x_{n-1}}{2(n-1)}$$

$$t_3 = \bar{x}$$

i) Show that each of 3 estimators is unbiased. (5 marks)

ii) Determine the efficiency of t_3 relative to t_2 (5 marks)

d) Let $x_1, x_2, .. x_n$ denote a random sample from a pdf is $P(X = x) = \frac{e^{-\mu}\mu^x}{x!}$, x=0,1,….

Use the factorization theorem to show that $T = \sum x_i$ is a sufficient estimator of μ

(4 marks)

e) Let $x_1, x_2, .. x_n$ be a random sample of size n from a distribution with mean from a population with mean μ and variance δ^2 , show that $\bar{x}$ is a consistent estimator of μ.

(5 marks)

f) Given a population with a distribution whose pdf is

$$\left[f(x,\theta) = \frac{1}{\theta}, 0 \leq x \leq \theta, 0 \; otherwise \right.$$

Using the method of moments obtain the estimator for θ. (4 marks)

QUESTION TWO (20 MARKS)

a) If $x_1, x_2, .. x_n$ is a random sample of size n from a population whose pdf is given by

$$f(x,\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, ...$$

Determine the maximum likelihood estimator (MLE) of λ. (8marks)

b) $Let\ x_1, x_2, .. x_n$be a random sample of size n from a population whose pdf is given by

$f(x,\theta) = \theta e^{-\theta x}$,x> 0,

Determine the Cramer-Rao lower bound for θ. (12marks)

QUESTION THREE (20 MARKS)

Three objects M, N and P with weights$t_{1,}t_2\ and, t_3$ respectively are weighed on a spring balance M and N together weigh 75g, M and P weigh 45g and N and P weigh 100g. Assuming the weights are independent with constant variance δ^2, determine the least square estimates of $t_1, t_2, and\ t_3$ (20 marks)

QUESTION FOUR (20 MARKS)

a) A random sample of size n is drawn from a normal population with $(0,\delta^2)$. Determine the minimum variance unbiased estimator (MVUE) of δ^2 and its variance. (10 marks)

b) Let $x_1, x_2, .. x_n$ be a random sample of size n from a population whose pdf if $f(x,\theta)$where θ is unknown. Let $T = t(x_1, x_2, ... x_n)$ be the necessary and sufficient condition for T to be the Minimum variance unbiased estimator (MVUE) of θ expressed in the form

$$\frac{\partial LogL}{\partial \theta} = \frac{T - \theta}{\lambda}$$

Show that T is MVUE of θ and $Var(T) = \lambda$ (10 marks)

QUESTION FIVE (20 MARKS)

a) Let $x_1, x_2, .. x_n$ denote a random sample from a pdf

$$f(x) = \begin{bmatrix} (\lambda + 1)x^{\lambda}, & 0 < \mathrm{x} < 1, \ \lambda > 0 \\ 0 & elsewhere \end{bmatrix}$$

Obtain the moment estimator of λ (8 marks)

b) Given a random sample of size n from a population whose pdf is

$$f(x) = \frac{1}{2\sqrt{2\pi\theta}} e^{\frac{-1}{4\theta}(x-5)^2}$$

Obtain the maximum likelihood estimator (MLE) of θ and show that it's unbiased. (12 marks)