



MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SECOND SEMESTER EXAMINATION FOR
BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

SUPPLEMENTARY EXAMINATION

ECU 107: ENGINEERING MATHEMATICS IV

DATE:

TIME:

INSTRUCTIONS

Answer question ONE (Compulsory) and any other TWO questions

QUESTION ONE (COMPULSORY) (20 MARKS)

a) Evaluate $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$ (3 marks)

b) Consider the function $f(x) = \begin{cases} \frac{x^2 - 1}{x - 2} & x \neq 2 \\ 0 & x = -2 \end{cases}$

i) Sketch the graph of the function $f(x)$ (2 marks)

ii) Determine the points of discontinuity(if any) of the function $f(x)$ (3 marks)

c) Prove that $\frac{d}{dx} (\sec x) = \sec x \tan x$ (3 marks)

- d) A projectile is fired straight upwards with a velocity of 400 m/s ; its distance above the ground t sec. after being fired is given by $S(t) = -16t^2 + 400t$. $S(t)$ is the distance of the particle from the ground after it has been fired. Calculate the maximum height achieved by the particle (3 marks)
- e) Evaluate $\int \sin(3x - 1) dx$ (3 marks)
- f) Determine Taylor's series expansion generated by the function $f(x) = \ln x$ upto the fifth term about $x = 2$ (4 marks)
- g) Calculate the volume obtained by rotating the area under the curve $y = 1 + x$ between $x = 1$ and $x = 2$ about the axis of x (4 marks)
- h) Show that improper integral $\int_1^{\infty} \frac{1}{x} dx$ is divergent (5 marks)

QUESTION TWO (20 MARKS)

Consider the function $f(x) = x^3 + x^2 - 5x - 5$

- i) Use the second derivative test to find the local maximum and minimum of the function $f(x)$ (7 marks)
- ii) Discuss the concavity of $f(x)$ (5 marks)
- iii) Determine the points of inflection of $f(x)$ (4 marks)
- iv) Sketch the graph of the function $f(x)$ (4 marks)

QUESTION THREE (20 MARKS)

- a) A vessel containing water is in the form of an inverted hollow cone with semi-vertical angle of 30° . There is a small hole at the vertex of the cone and the water is running out at a rate of 3cm^3 per second ($3\text{cm}^3 / \text{s}$). Calculate the rate at which the surface area in contact with water is changing when there is $81\pi \text{ cm}^3$ of water remaining in the cone. (6 marks)
- b) Evaluate $\int \frac{3x+7}{(x-1)(x^2+1)} dx$ (6 marks)
- c) Prove that $\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$ (5 marks)
- d) Calculate $\int e^{\sin x} \cos x dx$ (3 marks)

QUESTION FOUR (20 MARKS)

- a) Determine $\frac{dy}{dx}$ for;
- i) $y = \sinh x$ (2 marks)
- ii) $y = \tanh x$ (2 marks)
- iii) $y = \frac{(x^2+1)^5}{\sqrt{1+x}}$ (3 marks)
- b) Calculate $\frac{d^2y}{dx^2}$ if;
- i) $x = t - t^3$; $y = t - t^2$ (3 marks)
- ii) $x = \cos t$; $y = (1 - \sin^2 t)^{\frac{1}{2}}$ (3 marks)
- c) Calculate the gradient of the curve $x^3 + 2x^2y^2 - 2y + xy = 2$ at the point $(-4, 1)$ (7 marks)

QUESTION FIVE (20 MARKS)

a) Evaluate

i.) $\lim_{x \rightarrow 0^-} \frac{|x|}{x}$ (3 marks)

ii.) $\lim_{x \rightarrow 2^+} \frac{(x^2 + 2)|x|}{x^2}$ (3 marks)

b) Consider the function $f(x) = \begin{cases} 7x - 2 & x \geq 2 \\ 3x + 5 & x < 2 \end{cases}$

i) Evaluate $\lim_{x \rightarrow 2^-} f(x)$ (3 marks)

ii) Sketch the graph of the function $f(x)$ (3 marks)

c) Calculate the slope of the tangent line to the curve $x^3 + 2xy - 3y^2 = 9$ at the point $(3, 2)$ (4 marks)

d) If $xy + y^2 = 1$ find;

i.) $\frac{dy}{dx}$ (2 marks)

ii.) $\frac{d^2y}{dx^2}$ (2 marks)